THE UTILIZATION OF PROBABILISTIC CONTROLS IN A STANDARD COST SYSTEM

By

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CHAPTER I

INTRODUCTION

Standard costs have been typed as "benchmarks," "yardsticks," and "gauges." Regardless of the characterization employed, the key point is that standard costs are predetermined costs. As such, they are used in cost control, inventory costing, income determination, and pricing. However, a review of the literature in the area of cost accounting indicates general agreement that the principal function of standard costs is cost control.

Periodically, variances are isolated by comparing standard costs with costs actually incurred. These variances, in turn, can be further subdivided into several components. For example, cost accounting texts identify price and usage variances in connection with materials, rate and efficiency variances with labor, and spending, efficiency and volume variances with overhead.[1] Actual cost control is achieved by identifying and eliminating the causes of these variances. Recently, however, a number of deficiencies have been cited in connection with this procedure.

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[1]Michael Schiff and Lawrence J. Benninger, Cost Accounting (2nd ed.; New York: Ronald Press Co., 1963), Chapters 14 and 15.

Deficiencies of Traditional Cost Control Procedures

The primary objection is that all variances are reported to management without any indication of their significance.[2] This "deterministic"[3] approach implies that each variance is significant, and therefore requires an investigation and explanation. Because of time, manpower and monetary constraints, however, management must rely on judgment and experience in evaluating the significance of a particular variance. As a result, rules of thumb are often relied upon to identify those variances which require investigation. Examples of these rules include relating the variance to a subjectively predetermined dollar amount or percentage of standard cost. The deficiency of such criteria is that they ignore the possibilty that variances may be the result of purely "stochastic" elements.

Other deficiencies associated with the deterministic approach stem from the reliance upon periodic reports. The common practice of reporting variances at month end frequently leads to barren investigations. The greater the time lapse between the occurrence of a variance and its disclosure, the more difficult it becomes to isolate the reason for the variance.[4]

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[2]Charles T. Horngren, Cost Accounting, A Managerial Emphasis (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1967), p. 802.

[3]Z. S. Zannetos refers to the listing of all variances as "deterministic" because it does not recognize a distribution around the standard. See Z. S. Zannetos, "Mathematics as A Tool of Accounting Instruction and Research," The Accounting Review, Vol. XXXVIII (April, 1963), pp. 326-335; and "Standard Costs As a First Step to Probabilistic Control: A Theoretical Justification, An Extension and Implications," The Accounting Review, Vol. XXXIX (April, 1964), pp. 296-304.

[4]Horngren, op. cit., p. 802.

A related problem is that of aggregation. Because the performance report typically covers an extended time period such as a week or month, variances taking place during this time are frequently consolidated. Investigation decisions are then made on the basis of reported net variances. This, in turn, gives rise to the possibility that significant favorable and unfavorable variances will be offset and an opportunity for cost reduction overlooked. If, on the other hand, a net variance is considered significant, the problem of isolating the significant components of this variance remains.

Horngren summarizes the points as follows:

> This combination of delayed reporting and cost accumulations that represent a conglomeration of different operations makes it difficult to find causes for variances and to trace causes for them below the foreman level to individual machines, men and materials.[5]

Probabilistic Controls

In an effort to overcome these deficiencies, a number of writers in the recent accounting literature have recommended that probabilities be utilized in the formulation of a variance investigation decision rule.[6] The importance of the use of probabilities

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[5]*Ibid.*, p. 802.

[6]For example: Harold Bierman, Jr., Lawrence E. Fouraker and Robert K. Jaedicke, "A Use of Probability and Statistics in Performance Evaluation," *The Accounting Review*, Vol. XXXVI (July, 1961), pp. 409-417. F. S. Luh, "Controlled Cost: An Operational Concept and Statistical Approach to Standard Costing," *The Accounting Review*, Vol. XLIII (January, 1968), pp. 123-132. Mohamed Onsi, "Quantitative Models for Accounting Control," *The Accounting Review*, Vol. XLII (April, 1967), pp. 321-330.

lies in the fact that they provide an objective criterion which can be used to distinguish between significant and insignificant variances. The basis for this distinction is the explicit recognition of the role of "chance" in performance.

Experience has shown that the time required to perform a specific operation varies under what appear to be essentially similar conditions. Such variation is the result of a complex set of causes, no one of which can be isolated as being primarily responsible for the variation. Recognition of this concept leads to the realization that some variation in performance is inevitable, and that such variation need not be investigated. On the other hand, some variations in performance are not due to chance and can be traced to specific causes known as "assignable causes." By definition, these can be identified and eliminated.

The objective of probabilistic control is to distinguish between these two types of variation.[7] Only those variations stemming from assignable causes are considered significant and therefore in need of investigation. The opposite applies to chance variation. The significance determination is made by considering the probability that a given variation is the result of chance factors; hence, the term probabilistic control.

In addition to providing an objective significance criterion, probabilistic controls eliminate the other deficiencies noted as associated with the deterministic approach, namely, lack of timeliness and aggregation. These are eliminated because random

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[7]Robert W. Koehler, "The Relevance of Probability Statistics to Accounting Variance Control," Management Accounting, Vol. L (October, 1968), p. 37.

sampling during the production cycle is used as the vehicle for identifying problem areas. Such identification on a current basis provides the opportunity for current cost control.

Need for This Study and Its Purpose

The benefits to be derived from a recognition of the stochastic elements inherent in a productive process have been presented in the accounting literature. However, there are very few references in this literature to the actual utilization of probabilistic variance controls in connection with the current output of a productive operation.[8] This is supported by the findings of R. W. Koehler:

> In some general inquiry from some prominent corporations I was unable to find a single use of statistical procedures for variance control.[9]

Accordingly, the purpose of this study is twofold:

1. To explore the possibility of utilizing probabilities for cost control in the A. O. Smith Corporation, Milwaukee, Wisconsin;

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[8] It should be noted that there is a considerable body of literature concerning probabilistic controls stemming especially from the areas of statistics and engineering. This literature contains both theoretical expositions and reports on empirical studies. Up until recently, however, the stress of these writings was to apply probabilities in a narrow fashion to problems of control. For example, control charts were applied to the problems of defects, and if defects were within stipulated limits a situation under control was assumed. Although modern writers envisage a broader usage of probabilities, it is curious that Richard M. Duvall, writing in the June, 1967, Management Science on "Rules for Investigating Cost Variances," Vol. 13, pp. b631-b641, cites a number of accounting writers in support of his argumentation.

[9] Koehler, *op. cit.*, p. 35.

2. To test the feasibility of utilizing probabilistic standard costs in the A. O. Smith Corporation.

Methodology Employed

Because the basic assumption to be tested in this study is that a probabilistic model can be made operational, the cooperation of a manufacturing firm was a prerequisite. Consequently, the empirical portion of this study was conducted at the A. O. Smith Corporation, Milwaukee, Wisconsin. This corporation was selected because of its varied product line and diversified operations.

Initially, consideration was given to the data available for the development of a probabilistic model and the ability of the firm's cost control system to generate the data necessary for the continuing operation of the model. On the basis of this information a specific probabilistic control model was then selected for utilization. In order to obtain the input required by the model, daily random samples of actual labor cost were taken by the writer and the company budget analyst during November and December, 1968. Each sample observation consisted of the actual cost of performing the operation in question. This cost, in turn, was obtained in two steps. First, the automatic counter attached to the machine was checked at irregular intervals and a notation was made of the time taken to perform the operation on the required number of pieces. The actual labor cost was then read from a table prepared expressly for this purpose. In all, 568 such individual observations covering three operations were made during the course of the study. Closely associated with the empirical phase of this study was library

research conducted at the University of Florida and at Marquette University.

Plan of the Study

A description of the salient features of the A. O. Smith Corporation's traditional type of standard cost control system is presented in the following chapter. This inclusion serves as a base against which the results obtained from the probabilistic model can be compared.

Chapter III is devoted to a discussion of a number of probabilistic models suggested for use in cost control. Each such model is evaluated in terms of its potential for application to the A. O. Smith Corporation. On the basis of this evaluation a model was selected for implementation.

The actual implementation of the probabilistic model selected is described in Chapter IV. As such, it includes a discussion of the source of the data used in the formulation of the model, the problems encountered, the decisions made during implementation, and the results obtained.

A modification of the probabilistic model originally selected is presented in Chapter V. The primary reason for this alteration was to secure the support of the first-line supervisors. However, the revised model represents an alternative approach to cost control. Accordingly, the chapter concludes with a comparison of the results obtained from the model before and after its modification.

In Chapter VI selected criteria are identified and utilized to evaluate the probabilistic controls employed. In the process of evaluation each of the criteria is applied to both the traditional cost control system employed by the firm and the probabilistic controls. Finally, in Chapter VII, the conclusions emerging from this study are presented.

CHAPTER II

A. O. SMITH CORPORATION COST CONTROL SYSTEM

Operation of Traditional Controls

This chapter is concerned with the cost control system currently operated by the A. O. Smith Corporation and its ability to generate the data necessary for the formulation of a probabilistic cost control model. The plan of this chapter is first to describe the cost control environment of the company, and second to indicate its utilization by the writer in accomplishing his research.

The A. O. Smith Corporation is a diversified manufacturing firm which reported sales of $330 million during 1967.[1] The firm is organized into four operating groups: Contract Products, Consumer Products, Industrial Products, and International and Special Products.[2] Each group, in turn, is subdivided into operating divisions of which there are twelve in number.

Following several meetings with representatives of management, it was decided that the automotive division would be a suitable area for the introduction of probabilistic cost controls. The bulk of the division is located at the Milwaukee Works. As the name implies, the output of the division is limited to automotive

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[1]A. O. Smith Corporation, 1967 Annual Report.

[2]Ibid., p. 2.

components, including automobile frames and wheel suspension control arms.[3]

In this division several departmental performance reports are prepared on a weekly basis. At month end these reports are consolidated into a divisional performance report. A discussion and description of the system which generates data for these reports follow:

Standard Material Cost

The standard material cost applicable to a particular part number has four components. The first is the list price of the coil steel used in the part. This is based upon an engineering layout of the sheet from which the blank is drawn. Added to this is a proportionate share of the cost of freight-in. Then a deduction is made for the salvage value of the steel remaining after blanking. Finally, a charge is added for work done in the material preparation department. Depending on the part number involved, this may include the cost of decoiling the steel, cutting it to length, and pickling.

A material price variance is isolated at the time the material is purchased. Responsibility for this variance rests with the purchasing department. In addition, a material usage report is prepared each month. Included in this report is a usage variance computed for each part number. These variances, in turn, are combined and a net variance is presented for each product line. The final consolidation is at the divisional level.

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[3]*Ibid.*, p. 25.

Because each reported usage variance represented a consolidation of the previous month's activity, several management representatives were queried as to the criteria used to distinguish between significant and insignificant variances. The variety of responses to this question served to indicate the absence of previously specified criteria. Some supervisory personnel relied upon a comparison of current and past performance, while others were interested in the variance as a percentage of standard. Even those who cited percentage of standard as the applicable criterion failed to agree on what constituted a significant variance. In fact, the responses ranged from 2 per cent to 10 per cent of standard.

Standard Manufacturing Overhead Rates

The company calculates standard variable and fixed overhead rates each year. Both are applied on the basis of standard direct labor dollars earned.

Each year the departmental foreman, assisted by the budget analyst, estimates the variable expenses to be incurred by the department during the coming year. These estimates are based on the projected production schedule. A standard variable overhead rate is then calculated for each functional cost classification by dividing the estimated variable costs by estimated departmental direct labor dollars. When added together these individual rates yield a standard variable overhead rate which is used for product costing.

The standard fixed overhead rate is obtained by dividing estimated total fixed overhead by the total estimated direct labor dollars.

During the year a departmental overhead expense report is prepared each week. Included in this report are allowable expenses based on the level of activity attained as well as the actual expenses incurred. A variance is then reported for each cost classification.

At the end of each month, these individual variances are combined and a net departmental variance is reported. Finally, the net departmental variances are consolidated into a divisional overhead variance.

Upon investigation it became apparent that a problem similar to the one encountered in connection with material cost control existed with respect to overhead. This was the absence of a variance investigation decision rule. In fact, it appeared that only the monthly net departmental overhead variance was the object of the foreman's concern. The individual weekly variances were reviewed only if the monthly net variance exceeded some subjectively predetermined amount. A subsequent attempt to identify this amount proved to be fruitless.

Standard Direct Labor Cost and Related Computations

The focal point of standard direct labor cost is the "standard hour," which is defined as the allotted time for the production of a specified number of pieces.[4] The time per piece or per 100 pieces is established by the Industrial Engineering Department using work measurement procedures. This unit of measurement is central to

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[4]A. O. Smith Corporation, Factory Cost and Labor Controls, p. 3.

the system because it serves as the basis of employee compensation, labor controls, and the standard inventory rate.

To determine the earnings of an employee on a given operation the following information is required:

1. Work measurement standard: the allotted time stated in decimal parts of an hour per piece or per 100 pieces.
2. Base rate: the hourly rate expressed in dollars which is paid to the employee for each productive hour earned.

The actual computation of employee earnings is accomplished in two steps. Earned hours is the product of the work measurement standard and the number of pieces produced. Total earnings is the product of earned hours and the base rate. For example, if an operator produces 1,000 pieces and the work measurement standard is .5, then earned hours is equal to 1,000 x .5/100 or 5. Given a base rate of $2.00, the operator's earnings would be $2.00 x 5 or $10.00.

The basic payroll calculations can be modified to yield several measures which serve as controls. One of these controls is known as "Percent Performance."[5] By dividing earned hours by the hours spent on production, the productivity of an individual can be evaluated on those operations to which work measurement standards have been applied. If the operator in the preceding example had spent four hours on the operation, his per cent performance would have been 5/4 or 125 per cent.

Another measure used for control is "Percent on Measured Work."[6] By comparing clock hours with the time spent on measured

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[5]Ibid., p. 11.

[6]Ibid., p. 12

work, the utilization of productive employees can be evaluated. Closely related to per cent on measured work is the "Utilization Ratio,"[7] which compares clock hours with earned hours.

The frame of reference in each of the above measures is time. The other possibility is to measure performance in terms of dollars. Standard inventory rates are obtained by converting the work measurement standards into monetary units. These rates represent standard labor costs, by part number and operation.

A sample "performance report" portraying actual and standard labor costs, as well as variances, is given in Table 1. This report is prepared weekly for the preceding week and is distributed to the departmental foremen and budget analysts. It is the joint responsibility of the foreman and the budget analyst in each department to identify those operations which require investigation. Once again, however, individual variances are investigated only if the net departmental variance appears out of line. Here, too, the question of applicable criteria could not be resolved.

An additional report (not illustrated) is prepared monthly at the divisional level. The reported figure represents a consolidation of the net variances reported by each department in the division. In this case the relationship of the reported variance to that of the previous month appeared to be the primary consideration in evaluating its significance.

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[7] Ibid., p. 14

Table 1

Sample Labor Performance Report

Dept.	Number	Operation	Date	Shift	Quantity	Actual Cost	Standard Cost	Variance $	Variance %
4	12	14	11-18	1	3900	$127.45	$122.46	4.99	4.07
			19	1	1805	57.63	56.68	.95	1.68
			19	2	3100	96.06	97.34	1.28 <u>cr</u>	1.31 <u>cr</u>
			20	1	1600	52.80	50.24	2.56	5.10
			20	2	3160	101.66	99.22	2.44	2.46
			21	1	1604	47.59	50.34	2.78 <u>cr</u>	5.52 <u>cr</u>
			22	1	3600	116.30	113.07	3.26	2.88
			22	2	<u>4100</u>	<u>121.38</u>	<u>128.74</u>	<u>7.36</u> <u>cr</u>	<u>5.72</u> <u>cr</u>
		TOTAL			22,869	$720.87	$718.09	$2.78	3.87

Cost Element Selected for Study

If it is to be useful, a probabilistic control model should be applicable to as many costs as possible. Accordingly, any one of the cost elements previously discussed could have been selected for this study. The classification actually chosen was labor cost.

By virtue of this selection, attention was not restricted to a small number of departments as would have been the case had material cost been selected. Instead, it was possible to apply the model to a variety of operations. In addition, daily random samples of labor cost could readily be taken. Finally, the influence of chance factors on performance can be easily demonstrated in this context. As previously indicated, it is the behavior of these factors which serves as the basis for the formulation of a variance investigation decision rule.

Measure of Performance Selected for Study

After labor cost had been selected as the cost element of interest, it was still necessary to identify the measure of performance to be used in the probabilistic control model. In this connection several measures of performance could have been utilized. These include:

1. The actual time to perform an operation on a single piece or some multiple thereof,
2. The actual labor cost of performing an operation on a single piece or some multiple thereof,
3. The variance between actual labor performance and standard measured in terms of time,

4. The variance between actual and standard labor cost measured in dollars,
5. The percentage of variance using either a time or dollar base.

For the purpose of this study the actual labor cost of performance was selected. A primary factor responsible for this choice was the ready availability of the data necessary for the formulation of a probabilities control model. As a result, no major modification of the existing cost control system was necessary in order to implement probabilistic labor cost controls on a selective basis.

Another consideration was the familiarity of operating personnel with reports cast in monetary terms. Accordingly, the selection of actual labor cost required a minimum of employee orientation to the use of probabilistic controls. The importance of this factor will be enhanced at the time consideration is given to the adoption of probabilistic cost controls on a plant-wide basis. Furthermore, in so far as this study is concerned, it minimized to the extent possible demands on firm personnel.

Finally, actual labor cost provides the possibility of giving substance to an alternative interpretation of standard, that is, the definition of standard as the level of performance under conditions of control.[8] An outgrowth of the probabilistic model will be to

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[8]This approach was first brought to the writer's attention by Dr. L. J. Benninger of the University of Florida during a discussion of the various interpretations of the concept of standard. Dr. Benninger applies his concept to expected average performance under conditions of control for an annual fiscal period.

identify the standard cost of an operation as the mean of the actual labor cost under controlled conditions. This follows from the fact that one of the purposes of identifying the performance distribution associated with an operation is to delineate the meaning of conditions of control.

Operations Selected for Investigation

Initially, three departments were randomly selected from the group comprising the automotive division. Because departments represent a grouping of similar operations, more than one department was included. This provided the opportunity to apply the probabilistic model to a variety of operations. For purposes of identification these departments will simply be referred to as #1, #2, and #3.

One operation was then randomly selected from within each department. This was accomplished by numbering all the operations in each department and using a table of random numbers to make the selection.

The final delimitation involved the selection of part numbers associated with each operation. The random number table could not be utilized in this case because not all part numbers were being run at the time this study was being conducted. Therefore, with the aid of the production scheduling section, those parts which were being produced during November and which would be produced during December were isolated. Part numbers were then randomly selected from this list.

The most noteworthy aspect of the cost control system currently utilized by the A. O. Smith Corporation is the absence of

basic variance investigation decision rules. Weekly and monthly reports are prepared for each cost element; yet, as indicated earlier in the chapter, various rules of thumb are relied upon to determine those variances in need of investigation. This situation is further complicated by the fact that these rules vary with the individual involved. This was illustrated by the assertion of one supervisor who claimed a variance was significant if it exceeded 10 per cent of standard. A second supervisor cited 2 per cent as the cut-off point for the same cost element. This deficiency led to a consideration of probabilistic control models which provide decision rules. These models are discussed and analyzed in the next chapter.

CHAPTER III

ALTERNATIVE PROBABILISTIC CONTROL MODELS

Several probabilistic control models have been presented in the accounting literature. In each case the objective was to utilize probabilities in the formulation of variance investigation decision rules. These models were reviewed and given careful consideration in the course of this study. This chapter details the writer's analysis of them and presents his basis for the selection of one of them for use in experimentation at the A. O. Smith Corporation.

Control Chart

One of the first probabilistic models suggested for the purpose of cost control was the "control chart" originally developed in the area of quality control. While there are several types of control charts, each is based on an "objective" interpretation of probability. Therefore, before evaluating a control chart as a probabilistic model, the content of the objective interpretation will be considered.

Objective Interpretation of Probability

The theory of probability is a mathematical theory and, as such, is made up of a set of logical deductions from certain basic

assumptions.[1] This theory is concerned with the relationships among assigned probabilities as well as the consequences of these assignments and not with the selection of events and the method of assigning probabilities to the events selected.

> Whether or not a probability should be assigned to an event and how this probability may be assessed are empirical, not mathematical questions. As far as mathematics is concerned, the assignment of probabilities to the events of a sample space is arbitrary.[2]

According to the objective interpretation, probability is synonomous with long-run frequency. In this view probabilities may be assigned only to those events which are random in nature, and the probabilities assigned to such events are based on frequency ratios which converge on definite values.

> Objectivistic views hold that some repetitive events, such as tosses of a penny, prove to be in reasonably close agreement with the mathematical concept of independently repeated random events, all with the same probability. According to such views, evidence for the quality of agreement between the behavior of the repetitive event and the mathematical concept, and for the magnitude of the probability that applies (in case any does), is to be obtained by observation of some repetitions of the event, and from no other source whatsoever.[3]

The repetitive process (the tossing of the coin) which generates the observed event (the face of the coin) is known as a stochastic process. The observed event, in turn, is a stochastic,

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[1]Robert Schlaifer, Probability and Statistics for Business Decisions (New York: McGraw-Hill Book Co., Inc., 1959), p. 15.

[2]Chris A. Theodore, Applied Mathematics: An Introduction (Homewood, Illinois: Richard D. Irwin, Inc., 1965), p. 529.

[3]Leonard J. Savage, The Foundations of Statistics (New York: John Wiley & Sons, Inc., 1954), p. 3.

chance or random variable.[4] Thus, a stochastic process is a repetitive process "which generates outcomes which are not identical and not individually predictable with certainty, but which may be described in terms of relative frequencies."[5] It is these relative frequencies which become probabilities under the objective interpretation.

Flowing from this interpretation of probability is a particularized form of decision-making. In essence, this approach attempts to reach a decision "purely on the basis of the objective evidence, given certain prespecified risks of error that the individual is willing to accept."[6] However, the procedure is restricted by the available objective evidence.

> Since in the great majority of practical problems involving samples, the only long-run frequency distributions which are known with certainty are the conditional distributions of sample statistics given specified values of the parameters of the population from which the sample is drawn, this school developed procedures for reaching decisions under uncertainty by looking only at these conditional distributions.[7]

Control Chart as a Probabilistic Model

The control chart is a probabilistic control model based on the preceding statistical reference frame. It is a probabilistic

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[4]Samuel B. Richmond, Statistical Analysis (New York: The Ronald Press Company, 1964), p. 102.

[5]Ibid., p. 102.

[6]Jack Hirshleifer, "The Bayesian Approach to Statistical Decision, An Exposition," The Journal of Business, Vol. XXXIV (October, 1961), p. 472.

[7]Schlaifer, op. cit., pp. 606-607.

model because explicit consideration is given to the effect of chance factors on performance.

> Stated succinctly, chance affects some variance classifications; probability statistics evaluates patterns of chance occurrences; therefore, probability statistics is useful to analyze those variances affected by chance.[8]

In essence, the control chart seeks to distinguish between chance and assignable performance. In order to make this distinction, control limits are placed on both sides of a performance norm. Any measures of performance falling outside these limits are considered to be the result of assignable factors. Any variation within the established limits is attributed to chance. Thus, the term "control chart" refers to a visual representation of a variance investigation decision rule. However, the manner in which the performance norm and control limits are identified is not specified by the term "control chart."

In any event, it is necessary to specify the probability distribution associated with chance performance. Such an identification is necessary in view of the fact that subsequent inferences will be based on the laws of probability. However, the process of identification is complicated by the fact that the population of individual performance is infinite.

> This stems from the fact that we consider the population corresponding to a process to consist of all the outcomes that would be generated by the process if it operated indefinitely under the same conditions.[9]

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[8]R. W. Koehler, "The Relevance of Probability Statistics to Accounting Variance Control," Management Accounting, Vol. L (October, 1968), p. 37.

[9]John Neter and William Wasserman, Fundamental Statistics for Business Decisions (New York: McGraw-Hill Book Company, Inc., 1959), p. 607.

Several approaches to the identification of the probability distribution of chance performance have been suggested in the literature.

In a study made by Louis Tuzi,[10] 36 historical monthly variances from standard for each overhead account were tested for normality by means of the "chi-square" test. Having found that a number of these monthly variances formed a skewed distribution, he concluded that the use of control charts based on the normal distribution was inappropriate.[11] Using the same historical variances, the author then explored the possibility of using other distributions. In all, Tuzi enumerated four possibilities which could be used to specify the distributions applicable to the cost variances.

> The accounts under study are classified into one of the following four types of distribution: (1) normal, (2) bi-modal, (3) J-shaped or reversed J-shaped, and (4) skewed.[12]

These results are not at all unexpected when recognition is given to the fact that historical monthly variances were used in the computations. As such they represented the difference between actual overhead cost and the previously established standards. Accordingly, the direction of the skewness would depend upon management's philosophy at the time the standards were set. For example, if standard cost is established as the lowest cost that occurred in the past, a skewed distribution is a logical necessity.

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[10] Louis A. Tuzi, "Statistical and Economic Analysis of Cost Variances" (Ph.D. dissertation, Case Institute of Technology, 1964).

[11] Ibid., p. 107.

[12] Ibid., p. 133.

When consideration is given to adopting probabilistic controls on a plant-wide basis, several problems emerge. Under this approach the variances associated with each account would have to be tested to determine the applicable distribution. When this requirement is satisfied, the question of a procedure for establishing control limits still needs to be resolved.

The use of a control chart in connection with single time observations instead of monthly variances from standard was considered by Koehler.[13] Using a hypothetical example, he assumed the distribution of individual performances measured in terms of the actual time required to perform an operation to be normal in form.

> Normality is frequently assumed in statistical work; but it is rarely rigorously fulfilled. Since statistical decisions are based upon the laws of probability, inferences regarding the shape of a probability distribution are often necessary. If the shape of a given distribution does not differ significantly from normality, useful, although not precise, conclusions will result .... The assumption of normality is, therefore, a practical one if it provides useful results.[14]

The difficulty with this approach is its limited potential for industrial application. Because the analysis is based upon individual performance observations, the widespread assumption of normality is suspect. As a result, serious mistakes in inference could arise if this procedure were utilized in connection with a variety of industrial operations.

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[13]Robert Wallace Koehler, "An Evaluation of Conventional and Statistical Methods of Accounting Variance Control" (Ph. D. dissertation, Michigan State University, 1967).

[14]*Ibid.*, pp. 57-58.

Control Chart for Sample Means

The procedure employed in each of the cases cited can be improved upon by utilizing the experience gained in the area of quality control, for it was here that control charts were first developed by Walter A. Shewhart.[15]

The distinguishing characteristic of this approach is the use of sample statistics calculated during the productive process. Samples of a fixed size are taken and statistics of interest are computed for each sample. The statistics which are frequently utilized include the sample range and the sample mean.[16] Control charts can then be constructed for each of these performance measures.

Applicable Probability Distribution

The construction of a control chart for sample means is based upon the behavior of such means. If a large number of samples were taken from a productive operation, the individual means computed from the samples would form a frequency distribution of their own. In statistical work this distribution is known as the "sampling distribution of the mean." As is true of all frequency distributions, the sampling distribution of the mean has a unique central tendency and dispersion. Consequently, the sampling

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[15] Walter A. Shewhart, Economic Control of Quality of Manufactured Product (New York: D. Van Nostrand Co., Inc., 1931).

[16] Acheson J. Duncan, Quality Control and Industrial Statistics (3rd ed.: Homewood, Illinois: Richard D. Irwin, Inc., 1965), p. 338.

distribution applicable to a productive process can be specified by identifying the mean and standard deviation if it is normal.

Statistical theory provides some important insights regarding the relationship of the sampling distribution and the population from which the samples are drawn. In the long run, the average of the sample means will equal the average of the population. In addition, the standard deviation of the sampling distribution of the mean will be equal to the standard deviation of the universe divided by the square root of the sample size.[17]

The importance of the sampling distribution of the mean is its form.

> If the universe is normal, statistical theory tells us that the expected frequency distribution of the $\bar{X}$ (mean) values will also be normal .... This means that in sampling from a normal distribution that has a known average and standard deviation, statistical theory gives a complete picture of the expected pattern of variation of the averages of samples of any given size.[18]

But, as previously indicated, the widespread assumption of normality is suspect.

> It has already been stated that many observed distributions of industrial characteristics do correspond roughly to the normal curve. Nevertheless, many others do not. Serious mistakes are often made when it is assumed that the distribution of an industrial quality characteristic is necessarily normal.[19]

Accordingly, reference is made to statistical theory once again.

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[17]Eugene L. Grant, Statistical Quality Control (New York: McGraw-Hill Book Company, Inc., 1954), p. 97.

[18]Ibid., p. 98.

[19]Ibid., p. 100.

> Even where the universe is abnormal, the distribution of the sample means will usually approach a normal distribution if the sample is sufficiently large.[20]

The question then becomes the determination of what constitutes a sufficiently large sample. In this connection, the empirical work of Shewhart can be cited. Shewhart's experiment consisted of taking 1,000 samples of four from each of two distributions: one triangular and the other rectangular. A distribution was then prepared for each group of sample means. Even though the original distributions were decidedly abnormal, the two distributions of $\overline{X}$ values approximated the normal.[21]

> The main point to be noted here ... is that even with a great departure from normality in the universe, the distribution of $\overline{X}$ values with $n=4$ is approximately normal; in sampling from most distributions found in nature and industry, the distribution of $\overline{X}$ values will be even closer to normal.[22]

This is the advantage to be gained from using sample means in connection with control charts instead of variances or individual items. When sample means are used the cost analyst need not assume, or, for that matter, test whether the distribution of actual performance measures is normal when dealing with a specific manufacturing process or operation. Again Grant can be cited in this context:

> The great practical importance of the normal curve arises even more from its uses in sampling theory

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[20]Robert M. Trueblood and Richard M. Cyert, Sampling Techniques in Accounting (Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1957), p. 145.

[21]Shewhart, op. cit., p. 182.

[22]Grant, op. cit., p. 99.

> than from the fact that some observed distributions are described by it well enough for practical purposes. One may say with much more confidence that the distribution of averages of samples from an unknown universe will be close to normal than that the universe itself is normal.[23]

On the basis of the preceding reference frame, the normal distribution is utilized to approximate the probability distribution of chance performance associated with a particular productive operation. This distribution is specified upon estimation of the mean and standard deviation. These measures, in turn, are estimated from the data applicable to the operation.

Appropriate Control Limits

Once the mean and standard deviation of the applicable chance probability distribution have been estimated, the task of specifying appropriate control limits remains. The positioning of these limits determines the answer to a question which can be variously phrased:

> "Is there one universe from which these samples appear to come?" or "Do these figures indicate a stable pattern of variation?" or "Is this variation the result of a constant-cause system?" or merely "Do these measurements show statistical control?"[24]

A negative answer leads to a search for assignable causes, while a positive answer means no action will be taken. Thus, the selection of control limits is in effect the selection of the level of significance at which the hypothesis of control is to be tested.

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[23] Ibid., p. 100.

[24] Ibid., p. 101.

Whatever level is selected, there is always the possibility of error. One possibility is the investigation of chance variations. This is Type I error of statistical inference because, as indicated, no specific reasons can be identified in the case of chance variation. The magnitude of this type of error is governed directly by the level of significance selected. The opposite error, Type II, consists of the failure to investigate assignable performance. Because the magnitude of this error depends upon the value of the population mean, it cannot be directly evaluated. The problem of setting control limits is also complicated by the fact that the probability of committing one type of error can be reduced only by increasing the probability of incurring the other. Thus, in the formulation of a variance investigation decision rule it is necessary to strike a balance between the two errors of inference.

In the area of quality control, the control limits are placed a distance of 3 sigmas (i.e., standard errors of the mean) on either side of the mean of the sampling distribution. If the sample means are normally distributed, this includes 99.73 per cent of the means.

> To be specific, 68.27 per cent of the observations will be in the interval $\bar{\bar{X}} \pm 1$ sigma, 95.45 per cent in the interval $\bar{\bar{X}} \pm 2$ sigmas, and 99.73 per cent in the interval $\bar{\bar{X}} \pm 3$ sigmas.[25]

Thus, if only chance factors are operative (i.e., universe unchanged), approximately three sample means in a thousand will fall outside the control limits.

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[25]Charles T. Horngren, Cost Accounting, A Managerial Emphasis (Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1967), p. 807.

However, whether sigma or probability limits are selected, it is still necessary to attempt to balance the two kinds of errors. Grant asserts that experience indicates 3-sigma limits actually do strike such a balance.[26] This assertion, however, is difficult to test empirically by virtue of the fact that the probability of a Type II error depends upon the true mean of the universe. Accordingly, 3-sigma limits must be justified from a practical point of view.

> The real basis for the use of 3-sigma limits on control charts for variables in industrial quality control is experience that when closer limits, such as 2-sigma, are used, the control chart often gives indication of assignable causes of variation that simply cannot be found, whereas when 3-sigma limits are used and points fall out of control, a diligent search will usually disclose the assignable cause of variation.[27]

Evaluation of Control Chart Approach

The significant feature of the control chart approach is that it provides an objective criterion for evaluating the significance of reported variances. As indicated, the basis for this determination is the probability that a given variation in performance was due to chance factors.

In addition, several practical considerations favor the use of this approach. In the introduction of the technique, control charts can be utilized in connection with data already collected for some other purpose. Actual changes in the method of data collection

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[26]Grant, op. cit., pp. 101-102.

[27]Ibid., p. 118.

can be postponed until the charts have proven their utility to management. The result is a minimum interruption during the implementation phase.

Once the control charts have become operational, they can be revised on a regular basis. By the use of sample results obtained during the productive process, the central line on the chart and the control limits can be revised to give effect to the learning curve as well as the elimination of assignable causes. The net effect is to identify standard cost as the level of performance under conditions of control.

Another vital consideration is the possibility of using control charts in connection with most components of manufacturing cost. The approach as outlined earlier in the chapter is applicable to both material and labor costs. Only a slight modification is required in the case of overhead items. This stems from the fact that samples of these costs cannot be taken on a daily basis. However, by the expansion of the time horizon, control charts can be utilized. Whether this would mean weekly or monthly tests would depend on the magnitude of the costs involved. The key point, however, is the widespread applicability of the technique.

However, the control chart approach has been criticized on the grounds that a level of significance is arbitrarily selected in setting the control limits. The critics assert that other factors should be considered when establishing the limits. These include the cost of conducting an investigation and the cost of failing to identify an assignable cause.

Controlled Cost

Another probabilistic control model considered in this study was the "Controlled Cost" model developed by Luh.[28] In essence, the approach is to test the hypothesis that two samples were taken from the same universe. The decision whether or not to investigate a productive operation is then made on the basis of the results of the test.

Description of Model

As was true of the control chart approach, this model recognizes the variability inherent in a productive process. Accordingly,

> Controlled cost is defined as the probability distribution of a collection of input costs in physical units required for the production of a given unit of output when a process is under control.[29]

However, the universe of controlled performance is unknown. Therefore, controlled cost must be specified on the basis of a sample.

> Accordingly, the knowledge about the universe of controlled performance must be inferred from limited and selective observations of a sample, presumably from the universe of controlled performance.[30]

Once the probability distribution associated with controlled performance has been specified, a sample of actual performance is then taken and a probability distribution corresponding to the results of this sample is prepared. It is assumed that if both

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[28]F. S. Luh, "Controlled Cost: An Operational Concept and Statistical Approach to Standard Costing," The Accounting Review, Vol. XLIII (January, 1968), pp. 123-132.

[29]Ibid., p. 125.

[30]Ibid., p. 125.

samples came from the universe of controlled performance, the respective probability distributions will be approximately the same. If the model is to be useful, it must evaluate the significance of the difference between the two samples. As indicated, the procedure is to test the hypothesis that the two samples were from the same universe.

In this type of test three variables must be considered:

> 1. Sample size. The size of the two samples being tested.
>
> 2. Precision. The magnitude of the differences in the probability distribution of the two samples being tested ....
>
> 3. Reliability. The probability corresponding to the precision obtained from tables, i.e., the degree of assurance in stating that the two samples being tested are from the same universe.[31]

Thus, before evaluating actual performance management must specify the desired degree of reliability. This is necessary because two samples are bdng tested and, therefore, the conclusion as to whether both samples were from the same universe can be stated only with a certain degree of reliability, i. e., assurance.[32]

> From the desired assurance, the size of the sample used in establishing controlled cost, and the size of the sample of actual performance to be evaluated, a corresponding "allowable deviation" between the probability distribution of controlled cost and that of actual cost is either computed using a mathematical function, or merely extracted from a mathematical table.[33]

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[31]F. S. Luh, "Controlled Cost: An Operational Concept and Statistical Approach to Standard Costing" (Ph. D. dissertation, The Ohio State University, 1965), p. 34.

[32]*Ibid.*, p. 49.

[33]F. S. Luh, *The Accounting Review*, *op. cit.*, p. 128.

The decision whether to investigate an operation is then made by comparing the predetermined "allowable deviation" with the actual deviation between the two samples. In other words, when the actual deviation is less than the allowable deviation, the operation is considered to be under control.

These procedures can be summarized as follows:

1. Determine the desired reliability.
2. Determine the resulting precision as a basis for rejecting or accepting the hypothesis.
3. Compute the actual differences in the probability distributions of the two samples.
4. Compare 2 and 3.
5. Accept or reject the hypothesis.[34]

Thus, the decision to accept or reject the hypothesis is made on the basis of sample information. Accordingly, the possibility of error exists. Even if the operation is under control, it is possible for the probability distribution of actual cost to be greater than the allowable deviation from controlled cost because of chance factors. Because this condition calls for an investigation, a Type I error will have been made. On the other hand, chance factors can cause the probability distribution of actual cost to fall within the allowable deviation even though the operation is actually out of control. Since no investigation will be conducted in this case, the result is a Type II error.[35] As long as samples are used these errors cannot be eliminated.

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[34] F. S. Luh, (Ph. D. dissertation), op. cit., p. 35.

[35] F. S. Luh, The Accounting Review, op. cit., p. 129.

<u>Evaluation of Model</u>

Luh asserts that the controlled cost model offers several advantages. The first is a "realistic specification of controlled cost."[36] This stems from the fact that the probability distribution gives explicit recognition to variability in performance. As a result, unrealistic assumptions are avoided, and the possibility for a more thorough analysis of costs exists.

However, when consideration is given to the adoption of the model as a variance investigation decision rule, a number of problems arise. A crucial matter which must be resolved is the desired degree of reliability to be employed. Luh suggests that such factors as the type of operation, the availability of managerial time, the extent of competition, and the results of past evaluations be considered in setting the desired reliability.[37] The difficulty is that such qualitative factors do not lend themselves to quantification. The matter is further complicated by the inter-relationship of the degree of reliability and the two types of errors previously mentioned. Ideally, that degree of reliability which balances the two errors should be selected. Practically, however, the selection must be somewhat arbitrary.

Balance expected costs of errors?

When the desired reliability has been selected, the matter of identifying controlled cost remains. Luh warns that the period used to establish controlled cost must be carefully selected.[38]

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[36] <u>Ibid</u>., p. 130.

[37] F. S. Luh (Ph. D. dissertation), <u>op</u>. <u>cit</u>., pp. 51-53.

[38] <u>Ibid</u>., p. 75.

However, short of an investigation of all performance in the controlled cost sample, the presence of assignable causes cannot be discounted

Another point to be considered is the model's potential for application. As indicated, it is desirable that a probabilistic control model be applicable to as many cost classifications as possible. However, as presented, "the controlled cost system is generally applicable for evaluating repetitive operations in labor and material usage...."[39]

The question of sample size and sample frequency is also unresolved. In the illustrations presented by Luh, sample sizes vary between 20 and 400.

Finally, the model is subject to the same criticisms cited in connection with the control chart approach. In other words, the model does not explicitly consider the cost of investigating and not investigating performance.

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[39] Ibid., p. 91.

Bierman, Fouraker and Jaedicke Model

Description of Model

A cost control model which explicitly considers the cost of investigating a variance, the cost of not investigating a variance, and the probability distribution of chance performance was developed by Bierman, Fouraker, and Jaedicke.[41] The "investigate" or "do not investigate" decision is made on the basis of the expected cost associated with each act. The information necessary for the calculation of expected costs given the occurrence of an unfavorable variance is shown in Table 2.

The table indicates that there are two acts and two states. Associated with each act-state combination is a conditional cost. If an investigation is undertaken, the cost of this act is C regardless of the cause of the variance. If, on the other hand, no investigation is undertaken and state 1 prevails, the cost is zero. This stems from the fact that the variance was due to random causes which cannot be eliminated. Given state 2 and no investigation, the cost is L because assignable or controllable causes are being overlooked.

The expected cost of each act is then computed by multiplying the conditional costs of the acts by the conditional probabilities of the states and summing. The procedure is illustrated for the act "investigate" in Table 3.

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[41] H. Bierman, Jr., L. B. Fouraker, and R. K. Jaedicke, Quantitative Analysis for Business Decisions (Homewood, Illinois: Richard D. Irwin, Inc., 1961), pp. 108-124. See also: Harold Bierman, Jr., Topics in Cost Accounting and Decisions (New York: McGraw-Hill Book Company, 1963), pp. 15-23.

Table 2

Conditional Cost Table*

States	Acts: Investigate	Acts: Do Not Investigate	Conditional probabilities of states, given an occurrence of unfavorable variance
1	C	O	P
2	C	L	1 - P
Expected cost of acts	C	L (1 - P)	

where: State 1. The variance was caused by random, noncontrollable causes, and no action is required by management.

State 2. The variance was caused by factors over which management does have control, and action by management is desirable.

C is the cost of investigation.

L is a present value of the estimate of cost inefficiencies in the future which are avoidable.

*Bierman, Topics in Cost Accounting, p. 20.

Table 3*

Expected Cost of Act 1 "Investigate"

	Conditional Cost	Probability	Cost x Probability
State 1	C	P	PC
State 2	C	1 - P	(1-P)C
		Expected Cost of Act 1:	C

*Bierman, Topics in Cost Accounting, p. 20

The decision rule is to minimize the expected cost of the act. Therefore, if the expected cost of investigating is less than the expected cost of not investigating, an investigation is called for. In the opposite case no investigation should be undertaken.

In order to illustrate the operation of the model, Bierman presented the following example.[41] The budgeted cost for a year was \$10,000. This represented the expected value of the cost to be incurred and was used as the standard cost. It was further assumed that the cost was normally distributed with a standard deviation of \$6,000. Given that the actual cost for the year was \$13,000, "the probability of a cost variance as large as or larger than \$3,000, given an unfavorable variance, was 62 per cent."[42] This is the conditional probability P appearing in Table 2.

[41] Bierman, Topics in Cost Accounting, pp. 17-22.

[42] Ibid., p. 21.

To complete the analysis, dollar values were assigned to C and L. Bierman assumed the cost of conducting an investigation to be a constant, i. e., C = \$40. L was assumed to be \$9,000 because "L is equal to three times the cost variance (i.e., the inefficiency is expected to last for about four years, assuming a 10 per cent interest factor)."[43]

With this information the expected cost of each act can be computed. The variance should be investigated because the expected cost of investigating is \$40 (C=\$40) while the expected cost of not investigating is \$3,420 ($L(1-P)$).

Evaluation of Model

Conceptually, the Bierman, Fouraker, and Jaedicke model is appealing because it explicitly considers the cost associated with each act and incorporates a consistent decision rule, i.e., minimize expected cost.

However, as presented, the model is applied to year-end variances from standard. Thus, even if a variance is investigated, there is a strong possibility that the investigation of the variance will prove to be barren because of the time lapse between the incurrence of the variance and its investigation. If, on the other hand, the cause of the year-end variance is identified, it is too late to correct the condition as far as the year in question is concerned. If it is to reap the cost control potential inherent in the probabilistic approach, the model should not be applied annually to summary

[43] Ibid., p. 22.

expense classifications but rather to operations on a current basis. In this way assignable causes could be identified and eliminated during the productive process.

Another point to be considered is the feasibility of accurately estimating C and L called for in the model. The authors assumed that the cost of investigation was a constant amount. However, based on the writer's experience at the A. O. Smith Corporation, such an assumption is open to serious question. In studies made which will be described in the following chapter, it was found that the cost of conducting an investigation was dependent upon the reason for the variation in performance. By definition, the cause of random or chance performance cannot be identified no matter how diligent or lengthy the search. Accordingly, an investigation of such performance will be costly because an exhaustive search must be conducted before it may be safely assumed that random factors were responsible for the variation. Even if performance is non-random in nature, the cost of investigation does not remain constant. The investigations conducted at the A. O. Smith Corporation demonstrated that the cost of investigation is further dependent upon the specific assignable cause involved. The more obscure the explanation, the higher the cost of investigation and vice versa.

The difficulty encountered in an attempt to estimate L, the avoidable cost inefficiencies, is the inability to predict the length of time the assignable cause would have remained operative. Bierman assumed the off-standard condition would persist for four years. However, such an assumption is unwarranted when an attempt is made to apply the model at the operational level because either

the operator or the foreman can be expected to detect such a condition within a more reasonable period of time.

Modern Decision Theory Model

The final probabilistic control model considered was the "modern decision theory model" as presented by Onsi.[44] This model is identified as modern because it possesses two distinguishing characteristics.

The first such characteristic is the explicit use of subjective probabilities. Onsi equates this term with personal judgment.[45] As such, they are based on a combination of intuition, experience, and objective evidence, and are subject to change as more information is acquired.

The other feature is the distinctive approach of the model. Instead of formulating a hypothesis and testing it by means of conditional probabilities as is done in the more traditional approach to decision-making, the Onsi model begins with a subjective probability distribution for the unknown parameter which is being investigated.

> The subjective probability distribution describes the decision-maker's state of information or degree of belief as to the several different conceivable values that the unknown parameter may take.[46]

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[44] Mohamed Onsi, "Quantitative Models for Accounting Control," <u>The Accounting Review</u>, Vol. XLII (April, 1967), pp. 321-330.

[45] <u>Ibid</u>., p. 325.

[46] Hirshleifer, <u>op</u>. <u>cit</u>., p. 472.

These a priori probabilities comprise the prior distribution.[47] Depending on the circumstances, a decision can be made on the basis of this prior distribution, or alternatively, the decision may be delayed until additional information in the form of a sample is obtained. If a sample is taken, the a priori probabilities are revised on the basis of the sample information to obtain a posteriori probabilities.[48] Because Bayes' Theorem is used in the revision process, the approach is also called "Bayesian statistics."

> The approach as a whole is called Bayesian because of the crucial role played by Bayes' Theorem in indicating how a specified prior probability distribution, when combined with sample evidence, leads to a unique posterior distribution for the unknown parameter.[49]

The final step in the procedure is to use the revised probabilities in the calculation of the expected cost of each act.[50] The act with the lowest expected cost is then selected.

Description of Model

Onsi used a highly automated process to illustrate the operation of the model.[51] The unknown parameter is the proportion (P) of defective units that will be produced by the process. The possible values which (P) may assume are shown in Column 1 of

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[47]Onsi, op. cit., p. 325.

[48]Ibid., p. 327.

[49]Hirshleifer, op. cit., p. 472.

[50]Onsi points out that the decision can also be made on the basis of expected opportunity loss. See pp. 327-329.

[51]Onsi, op. cit., pp. 326-329.

Table 4. Since it is assumed that each production run consists of 200 units, it is possible to have as few as two defectives on a single run (P =.01) or as many as thirty (P =.15).

Table 4

Unconditional Expected Costs*

(P) of Deviations	A Priori Probability	Cost of Acceptance Conditional	Cost of Acceptance Expected	Cost of Rejection Conditional	Cost of Rejection Expected
.01	.5	$ 1.00	$ .50	$4.00	$2.00
.05	.2	5.00	1.00	4.00	.80
.10	.2	10.00	2.00	4.00	.80
.15	.1	15.00	1.50	4.00	.40
	1.0		$5.00		$4.00

*Onsi, Quantitative Models, p. 327.

Prior to an actual run, (P) is unknown. If the run is allowed to proceed, i.e., accept the process as being under control, and P =.01, then two defectives will be produced. Since it is also assumed that the cost of reworking a defective is $.50, the conditional cost of acceptance is $1.00 (2 x $.50). In this way the conditional cost of acceptance for each value of (P) can be determined as shown in column 3 of Table 4.

Instead of accepting the process as under control, the alternative is to investigate the process before the run is allowed to begin. In this event two additional assumptions are needed. First,

the cost of investigation will always be \$3.00. Second, if an investigation is undertaken (P) will always be .01. As a result, the conditional cost of rejection (i.e., investigation) is \$4.00 for each value of (P). This information appears in column 5 of Table 4.

Before a decision can be made, subjective probabilities must be assigned to each possible event (value of P). Onsi asserts that these probabilities should be based on past records and experience.[52]

With this added information, the expected cost of each act can be computed by multiplying the conditional costs of the acts by their respective probabilities and summing for all possible events. The result is shown in columns 4 and 6 of Table 4.

At this point the process could be rejected because that act has the lowest expected cost. However, another possibility is to postpone the decision until a sample has been taken. By means of Bayes' Theorem, the sample results can then be used to revise the *a priori* probabilities. Finally, the expected cost of each act is recalculated on the basis of these revised probabilities. As shown in Table 5, the expected cost of accepting the process is less than the expected cost of rejection. Accordingly, acceptance is the best act under the circumstances.

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[52]*Ibid.*, p. 327.

Table 5

A Posteriori Unconditional Expected Costs*

(P) of Deviations	A Posteriori Probability	Cost of Acceptance Conditional	Cost of Acceptance Expected	Cost of Rejection Conditional	Cost of Rejection Expected
.01	.804	$ 1.00	$ .804	$4.00	$3.216
.05	.141	5.00	.705	4.00	.564
.10	.048	10.00	.480	4.00	.192
.15	.007	15.00	.105	4.00	.028
	1.000		$2.094		$4.00

*Onsi, Quantitative Models, p. 328.

Evaluation of Model

As indicated, the approach of the Onsi model is unique. This can best be illustrated by comparing it with the Bierman, Fouraker, and Jaedicke model. In each model the investigation decision is made on the basis of the expected cost associated with each act. The decision rule is the minimization of expected costs. However, the manner in which probabilities are utilized within the respective models is different. In the Bierman et al. model the probabilities refer to the likelihood of obtaining chance deviations as large as or larger than those observed. (This corresponds to the way probabilities are used in the control chart approach.) In the Onsi model, however, the probabilities refer to the likelihood that the unknown parameter (P) will assume a specific value.

By virtue of this modification, the Onsi model eliminates the primary objection to the control chart approach; namely, the arbitrary selection of the level of significance. However, it assumes that a mutually exclusive and collectively exhaustive listing of all possible events (parameters) can be made.

In the example presented there were four possible events, i. e., the unknown parameter (P) could assume four possible values. Because the example dealt with an automated process, it is conceivable that these values correspond to the care exercised during the set-up period. However, if Onsi's model were to be utilized in connection with labor cost it would be necessary to identify not only the probability distribution of chance performance but also the distribution associated with each assignable cause. The model presupposes the identification of all possible parameters. In the case of labor performance this would be average labor cost; but average labor cost during a particular period depends upon the nature of the causes operative during that period. Accordingly, the first requirement of the model is the identification of the distribution of values associated with each assignable cause. The mean of each such distribution would then represent a possible parameter.

This task is complicated by two factors. The first of these is the feasibility of obtaining a collectively exhaustive list of possible events. When the departmental foremen at the A. O. Smith Corporation were requested to prepare a list of possible assignable causes, they were unable to do so. However, they were able to identify the more common ones, but these were responsible for only about 50 per cent of assignable performance.

Even if such a listing could have been obtained, the problem of identifying the distribution of values associated with each cause remains. The difficulty is that of overlapping distributions. Koehler used the time required to shave to illustrate the problem.[53] He demonstrated that "the distribution of values due to chance overlaps the distribution of values due to assignable causes."[54] Chance and assignable causes operate simultaneously with the result that there are some common values associated with each cause. Only by a thorough investigation of these common performance values can the cause or causes be identified.

The final point to be considered in connection with the Onsi model is the specification of the conditional costs associated with each act-event combination. In the case of acceptance ("do not investigate"), Onsi relied upon a constant cost of reworking defective units in order to idéntify the various conditional costs. In other applications it would be necessary to estimate the loss associated with the failure to identify an assignable cause. The problems inherent in making such an estimate have been previously discussed in connection with the Bierman _et al._ model. Furthermore, Onsi assumed a constant investigation cost in specifying the conditional costs of rejection ("investigate"); yet the experience at the A. O. Smith Corporation substantiated that this cost is dependent upon the cause involved.

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[53]R. W. Koehler, _op. cit._, pp. 35-41.

[54]_Ibid._, p. 37.

Model Selected for Utilization

The probabilistic control model selected for application in this study was the control chart for sample means. This model incorporates probabilities into the standard cost system in order to formulate a decision rule to be used in connection with variance analysis. In this way the deterministic approach to variance reporting, which isolates historical variations, is replaced by a probabilistic technique which has the advantage of identifying problem areas on a current basis, thereby providing a means of current cost control.

More specifically, this model was selected because it is applicable to most categories of manufacturing costs. Its sole requirement is that the specific cost classification be influenced by chance factors. As such, the model is applicable to "variation in labor and overhead efficiency, material usage and overhead volume variances."[55]

Finally, and perhaps more importantly, this selection was based on the conviction that the control chart approach is the logical starting point in an attempt to utilize probabilistic cost controls. The numerous problems encountered in estimating the variables called for in the other models have already been indicated. The only possibility in such cases is to make a number of simplifying assumptions; nevertheless, the utility of the models depends upon the validity of the ensuing estimates.

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[55] *Ibid.*, p. 36.

Thus, in addition to providing a variance investigation decision rule, which is a desirable end in itself, the control chart approach will provide some of the information necessary to estimate those variables which play a crucial role in the other models. For example, the assignable causes at work in a particular operation as well as the value of the parameter associated with those causes will very likely be identifiable after a sufficient amount of experience has been acquired with the control charts.

Closely related to this point is the possibility of using the control chart as an evolutionary vehicle. Various aspects of the approach can be modified as additional information is acquired. One such possibility is to utilize the probability distribution of chance performance specified in connection with the control chart to calculate the expected cost of each act as called for in the Bierman _et al._ model. Another possibility is to use this same distribution as the "controlled cost distribution."

Finally, as far as the assertion that control limits are arbitrarily placed on the control charts is concerned, it is possible to place such limits at various distances from the central line. In this way the determination can be made as to which limits actually strike a workable balance between the two types of error.

CHAPTER IV

UTILIZATION OF CONTROL CHARTS FOR COST CONTROL

As indicated in the preceding chapter, the probabilistic cost control model selected for application in this study was the control chart for sample means ($\overline{X}$ chart). In this chapter a $\overline{X}$ chart for labor cost control is separately developed for each of the operations selected at the A. O. Smith Corporation: blanking, welding, and assembly. Since the purpose of each $\overline{X}$ chart is to isolate on a current basis those variations in performance which are probabilistically significant, it was necessary to estimate the probability distribution of chance performance associated with each operation. This was accomplished by means of 25 random samples of labor cost taken during November, 1968. From this sample information the mean (μ) and standard deviation (σ) of the universe of individual random performance were estimated. These parameters were then used to estimate the mean ($\overline{X}'$) and standard deviation ($\sigma_{\overline{X}}$) of the applicable sampling distribution. The predictive ability of the $\overline{X}$ charts derived from these latter estimates was subsequently tested during December, 1968, by means of additional samples of labor cost. However, before the construction of the individual $\overline{X}$ charts is described, several factors which apply to all three operations will be considered.

Preliminary Considerations

The universe of individual random performance associated with each operation refers to the distribution of labor cost values when only chance factors are operative. This is the process universe from which the samples are drawn. In order to estimate the parameters of this distribution (μ and σ), a number of samples were taken at the A. O. Smith Corporation during November, 1968. Each sample observation was taken during the productive process and represented the actual labor cost of performing the operation on a specific part number. Each sample, in turn, consisted of four observations from which a sample mean was computed.

Because the basis for subgrouping was the order of production, each sample consisted of items produced consecutively.

> Generally speaking, subgroups should be selected in a way that makes each subgroup as homogeneous as possible and that gives the maximum opportunity for variation from one subgroup to another. As applied to control charts on production, this means that it is of vital importance not to lose track of the order of production.[1]

The reason homogeneous subgroups are desired stems from the fact that "the variability <u>within</u> samples is used to deduce the variability <u>among</u> samples."[2] This refers to the use of the range (R) in the samples to estimate the standard deviation ($\sigma_{\bar{X}}$) of the sampling distribution.

A closely related point is the use of four observations in each sample. As indicated, the variability within samples is used

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[1]Eugene L. Grant, <u>Statistical Quality Control</u> (New York: McGraw-Hill Book Company, Inc., 1954), p. 151.

[2]W. A. Wallis and H. V. Roberts, <u>Statistics: A New Approach</u> (Glencoe, Illinois: The Free Press, 1956), p. 500.

to estimate the dispersion in the applicable probability distribution. This requires that there be a "minimum opportunity for variation within a subgroup."[3] Therefore,

> ... it is desirable that subgroups be as small as possible. On the other hand, a size of four is better than three or two on statistical grounds; the distribution of $\overline{X}$ is nearly normal for subgroups of four or more even though the samples are taken from a nonnormal universe....[4]

Thus, the relationship between the basis for subgrouping and the use of four observations can be summarized as follows:

> If observations are grouped on a "rational" basis ... the average of the variability within groups provides an appropriate measure of the variability to be expected between groups if the process is in control.[5]

Another consideration is the number of sample means required to specify the applicable distribution. While there appear to be few hard and fast rules in this area, at least one author has suggested the use of 25 samples.[6] After the $\overline{X}$ chart has been constructed, the estimates based on these initial samples may be revised as more sample information becomes available.

> The principal thing to be kept in mind is, however, that the main purpose of such a criterion is to detect lack of control in a continuous production process where we have a whole series of samples so that the question as to the minimum number of sub-samples becomes of minor importance.[7]

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[3]Grant, op. cit., p. 152.

[4]Ibid., p. 152.

[5]Wallis and Roberts, op. cit., p. 499.

[6]Grant, op. cit., p. 156.

[7]W. A. Shewhart, Economic Control of Quality of Manufactured Product (New York: D. Van Nostrand Company, Inc., 1931), p. 315.

In quality control applications the control chart for means is sometimes supplemented by a control chart for sample ranges (R chart). The purpose of "R Charts" is the control of process variability. However, the observation has been made that

> With fixed control limits, a change in M (process average) will be equivalent to a vertical shift of the normal distribution; this will increase the probability of a point falling outside the limits .... An increase in (σ) (process standard deviation) will also increase this probability, and a decrease in (σ) will decrease it. Thus the control chart for averages tends to detect process changes that involve changes in the mean or increases in the variability....[8]

The only change which the $\bar{X}$ Chart will not detect is a decrease in process variability.[9] However, decreases in variability will be recognized if the control charts are periodically revised. Accordingly, the R chart is not a strict necessity, and "the clerical costs of keeping R charts sometimes may be avoided without much loss in useful data."[10]

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[8]Wallis and Roberts, op. cit., p. 499.

[9]Ibid., p. 500.

[10]Charles T. Horngren, Cost Accounting, A Managerial Emphasis (Englewood Cliffs, N. J.: Prentice-Hall Inc., 1967), p. 811.

$\overline{X}$ Chart for the Blanking Operation

Construction of $\overline{X}$ Chart

In order to construct a $\overline{X}$ chart for the blanking operation it was necessary to specify the applicable sampling distribution of the mean. However, the parameters of this distribution are dependent upon the distribution of individual performance. Accordingly, the November sample information was used to estimate the parameters (μ and σ) of this latter distribution.

The means of the 25 samples taken from the blanking operation during November are shown in Table 6. Each observation represented the actual labor cost of blanking 100 pieces of part #207 in department #1. The mean ($\overline{\overline{X}}$) of the sample means ($\overline{X}$) was used as a preliminary estimate of μ . This estimate was obtained as follows:

$$\overline{\overline{X}} = \frac{\sum \overline{X}}{k} = \frac{\$98.28}{25} = \$3.93$$

where k is the number of sample means. The same value ($3.93) was used as the preliminary estimate of the mean of the sampling distribution because:

> The expected value, or mean of the entire probability distribution of $\overline{X}$-values is μ .[11]
>
> $$E(\overline{X}) = \mu$$

On the strength of this relationship, $3.93 was used as the central line on the preliminary $\overline{X}$ chart.

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[11]F. E. Croxton and D. J. Cowden, <u>Practical Business Statistics</u>(Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1960), p. 302.

Table 6

Means and Ranges of 25 Samples of 4 Taken from Blanking Operation during November, 1968, Part #207, Department #1

Sample Number	Sample Mean ($\overline{X}$)	Sample Range (R)
1	\$4.01	\$0.23
2	3.94	.20
3	3.87	.13
4	3.91	.16
5	3.88	.18
6	3.83	.13
7	3.87	.16
8	4.08	.22
9	3.92	.19
10	3.84	.09
11	3.86	.16
12	4.46	.66
13	3.87	.17
14	3.84	.14
15	3.85	.19
16	4.00	.17
17	3.85	.18
18	3.88	.14
19	3.93	.16
20	4.01	.21
21	3.91	.17
22	3.95	.18
23	3.88	.13
24	3.85	.09
25	3.99	.16
	$\overline{\overline{X}}$ = \$3.93	$\overline{R}$ = \$0.184

In order to set the control limits on the preliminary $\overline{X}$ chart it was necessary to estimate $\sigma_{\overline{X}}$. However, this measure of dispersion in the sampling distribution of the mean is dependent upon both the sample size (n) and the standard deviation (σ) of the universe from which the samples were drawn. This relationship is expressed as follows:[12]

$$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}}$$

Thus, before estimating $\sigma_{\overline{X}}$ it is necessary to estimate σ.

There are several ways of obtaining unbiased estimates of σ including the use of the sample range,[13] where range is the difference between the extreme observations within each sample.

> An unbiased estimate of σ can also be obtained from the range R.
>
> $$E\left(\frac{R}{d_2}\right) = \sigma \text{ .}^{14}$$

Values of d_2 have been computed and are available in published tables.[15] Croxton points out that:

> In quality control work it is a common procedure to compute R from a number of small samples, obtain the mean $\overline{R}$ of these values, and compute
>
> $$\hat{\sigma} = \frac{\overline{R}}{d_2} \text{ .}^{16}$$

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[12]*Ibid.*, p. 302.

[13]*Ibid.*, pp. 254-255.

[14]Dudley J. Cowden, *Statistical Methods in Quality Control* (Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1957), p. 63.

[15]*Ibid.*, p. 691.

[16]*Ibid.*, p. 63.

This estimate of the standard deviation of the process universe (σ) may be used to estimate the standard deviation of the sampling distribution ($\sigma_{\bar{X}}$) using the previously cited relationship

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

With this estimate, the control limits may be placed a distance of $3\sigma_{\bar{X}}$ on either side of the mean ($\bar{\bar{X}}$). Actual figures were not included in the preceding discussion because the procedure may be condensed into one step. Available tables permit the calculation of $3\sigma_{\bar{X}}$ control limits directly from $\bar{R}$.

> The two steps in the calculation of $3\sigma_{\bar{X}}$ might be consolidated as
>
> $$3\sigma_{\bar{X}} = \frac{3\bar{R}}{d_2\sqrt{n}}$$
>
> To shorten the calculation of control limits from $\bar{R}$, this factor $3 / d_2\sqrt{n}$, the multiplyer of $\bar{R}$ in the preceding calculation, has been computed for each value of n from 2 to 20 and tabulated .... This factor is designated A_2.[17]

By means of the A_2 factor, the $3\sigma_{\bar{X}}$ control limits may be calculated as follows:

$$\text{Upper Control Limit} = \bar{\bar{X}} + A_2\bar{R}$$

$$\text{Lower Control Limit} = \bar{\bar{X}} - A_2\bar{R}.$$

The range of each of the 25 samples taken from the blanking operation during November appears in Table 6. The average range ($\bar{R}$) was computed as follows:

$$\bar{R} = \frac{\Sigma R}{k} = \frac{\$4.60}{25} = \$0.184$$

where k is the number of samples.

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[17]Grant, op. cit., p. 104.

The control limits were then calculated by means of the A_2 factor and the average range ($\bar{R}$):

$$UCL = \$3.93 + .73\ (\$0.184) = \$4.064$$

$$LCL = \$3.93 - .73\ (\$0.184) = \$3.796$$

With this information the preliminary $\bar{\bar{X}}$ chart shown in Figure 1 was constructed. If the sample means are normally distributed, 99.73 per cent will fall in this interval.

Following the construction of the $\bar{\bar{X}}$ chart for the blanking operation, the sample means obtained during November were plotted on the chart. It is apparent from Figure 1 that the means of samples No. 8 and No. 12 fell outside the control limits. If the preliminary estimates were valid, it is very unlikely that these two sample values were due to chance factors because "only about three in a thousand will fall outside the control limits as long as the universe does not change."[18] Therefore, an investigation was undertaken in an attempt to identify the assignable causes responsible for these "out-of-control" points.

The initial step took the form of a meeting with the department foreman and budget analyst at which time the shifts associated with the respective sample means were isolated. Subsequently, an examination of the work tickets for the appropriate shift indicated that sample No. 8 included the production of an operator borrowed from another department. The extreme value associated with sample No. 12, on the other hand, was found to be the result of a hydraulic problem on the press, a fact which was substantiated by an examination of the service department report.

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[18]Horngren, op. cit., p. 807.

Preliminary Control Chart for Means of 25 Samples of 4 Taken
from Blanking Operation during November, 1968,
Part #207, Department #1

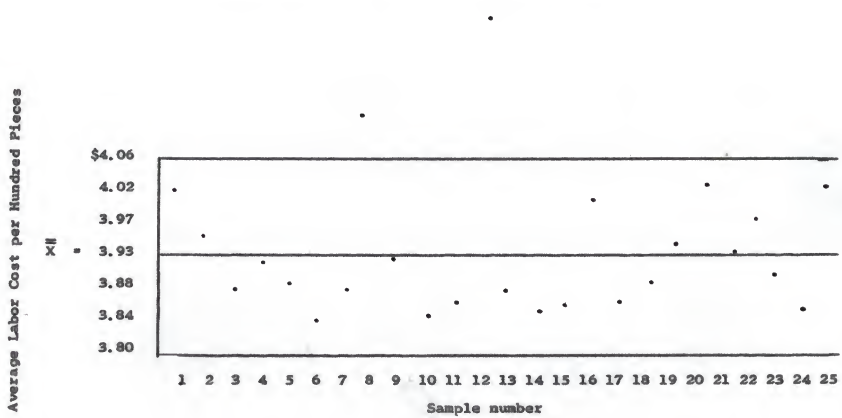

Figure 1

On the basis of these findings, it was concluded that the results of the two samples could not be attributed to chance. Accordingly, the means associated with these samples were dropped, and replacement samples were added in order to construct a second $\overline{X}$ chart. Table 7 includes the means and ranges of this revised group of samples.

The revised estimate of the mean of the sampling distribution was \$3.91, which again was the mean of the sample means ($\overline{\overline{X}}$). As above, this figure was used as the central line on the revised $\overline{X}$ chart.

The average range was used to construct the control limits. When the data of Table 7 are substituted, the revised control limits were placed at

$$UCL = \$3.91 + .73\ (\$0.16) = \$4.027$$

$$LCL = \$3.91 - .73\ (\$0.16) = \$3.793$$

As shown in Figure 2 all the sample means fall within the revised control limits. Hence, it was assumed that all sample observations were taken from a single universe. This must be an assumption because there is always the possibility of overlapping distributions. Thus, some of the values of chance performance coincide with values due to assignable causes.[19] However, the inability to identify the distribution associated with each assignable cause makes the hypothesis of a single universe reasonable for the purpose of cost control.

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[19]R. W. Koehler, "The Relevance of Probability Statistics to Accounting Variance Control," Management Accounting, Vol. L (October, 1968), p. 37.

Table 7

Means and Ranges of 25 Samples of 4 Taken from Blanking Operation during November, 1968, Part #207, Department #1

Samples 8 and 12 Replaced

Sample Number	Sample Mean ($\bar{X}$)	Sample Range (R)
1	\$4.01	\$0.23
2	3.94	.20
3	3.87	.13
4	3.91	.16
5	3.88	.18
6	3.83	.13
7	3.87	.16
8	3.96	.15
9	3.92	.19
10	3.84	.09
11	3.86	.16
12	3.98	.11
13	3.87	.17
14	3.84	.14
15	3.85	.19
16	4.00	.17
17	3.85	.18
18	3.88	.14
19	3.93	.16
20	4.01	.21
21	3.91	.17
22	3.95	.18
23	3.88	.13
24	3.85	.09
25	3.99	.16
	$\bar{\bar{X}}$ = \$3.91	$\bar{R}$ = \$0.16

Revised Control Chart for Means of 25 Samples of 4 Taken
from Blanking Operations during November, 1968,
Part #207, Department #1

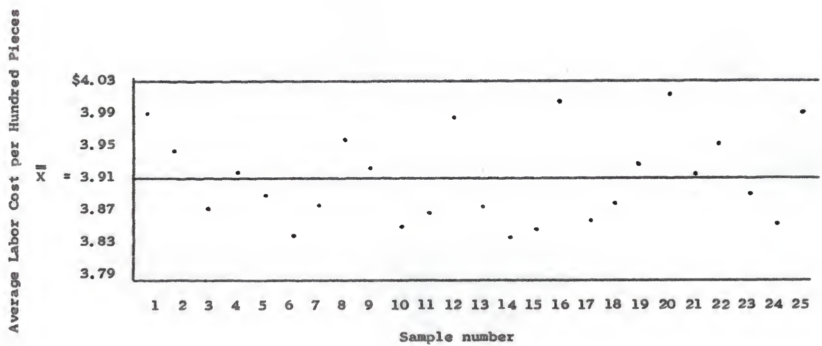

Figure 2

> If no points fall outside the control limits and if there is no evidence of nonrandom variation within the limits, it does not mean that assignable causes are not present. It simply means that the hypothesis that chance causes are alone at work is a tenable hypothesis and that it is likely to be unprofitable to look for special assignable causes.[20]

Test of $\bar{X}$ Chart

The samples taken during November, as well as the investigations conducted during that period, resulted in a revised $\bar{X}$ chart. In order to test the ability of the $\bar{X}$ chart to predict the pattern of random variation associated with the blanking operation, 20 additional samples were taken during the month of December, 1968. As before, each sample consisted of four observations taken during a single shift. Each observation, in turn, consisted of the actual labor cost of blanking 100 pieces of part #207 in Department #1. The mean of the four observations was then computed and plotted on the control chart reproduced in Figure 3.

The variation in performance occurring within the control limits was considered the result of chance. Accordingly, these points, with a few exceptions to be discussed later, were not investigated. However, a search for assignable causes was initiated immediately in connection with those points falling outside the limits.

The first such point was associated with sample No. 9. An investigation by the foreman revealed that blanks were occasionally

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[20]Acheson J. Duncan, Quality Control and Industrial Statistics (Homewood, Illinois: Richard D. Irwin, Inc., 1965), p. 341.

Control Chart for Means of 20 Samples of 4 Taken
from Blanking Operation during December, 1968,
Part #207, Department #1

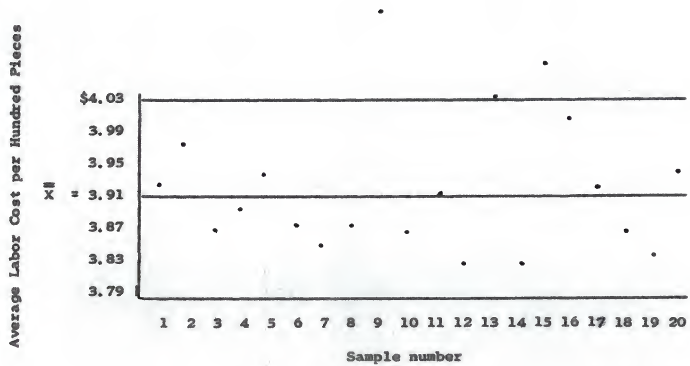

Figure 3

sticking in the die. This necessitated a departure from the standard operating procedure, thereby increasing the actual labor cost.

Figure 3 shows that the second point falling outside the control limits was sample No. 15. Again an investigation was undertaken by the foreman. In this case rusty steel stock was identified as the assignable cause.

In addition to the points falling outside the control limits, samples No. 2, No. 13 and No. 16 were investigated. The reason for this was to test further the 3-sigma control limits selected for this study. These control limits were selected because

> Experience indicates that in most cases 3-sigma limits do actually strike a satisfactory economic balance between the two kinds of errors.[21]

If this is true, it should be possible to isolate an assignable cause for the majority of points outside the control limits. This was accomplished in connection with sample No. 9 and sample No. 15. However, the opposite should be true for the points inside the limits. Accordingly, three of the more extreme points within the control limits were investigated.

Several points of interest emerged as a result of this activity. One was the excessive length of time spent investigating the results of these samples. In the case of samples No. 9 and No. 15, the reasons for the variations were readily identified. This was not surprising in view of the fact that the sample means represented a substantial departure from the estimated mean. However, this provided a marked contrast to the investigations conducted in connection

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[21]Grant, <u>op. cit.</u>, pp. 101-102.

with samples No. 2 and No. 13. Here an exhaustive search failed to identify anything resembling an assignable cause.

A different conclusion was reached with respect to sample No. 16. After a check had been made with the industrial engineering section, it was discovered that the operation was being run on a machine other than the one used when the job was originally timed. This result was totally unexpected in view of the fact that no explanation could be found for sample No. 13, which represented a greater variation in performance.

Another point worth noting is the absence of any sample falling below the lower control limit. However, this was expected because, short of a methods change, human endurance becomes the limiting factor.

$\overline{\overline{X}}$ Chart for the Welding Operation

Construction of $\overline{\overline{X}}$ Chart

The welding operation was the second of three operations at the A. O. Smith Corporation for which a $\overline{\overline{X}}$ chart was constructed. The procedure employed paralleled that described in connection with the blanking operation. In essence, the results of random samples were used to estimate the parameters of the applicable sampling distribution. These parameters were then used to construct the $\overline{\overline{X}}$ chart.

The means and ranges of the 25 random samples taken from the welding operation during November, 1968, are shown in Table 8. Each sample mean represents the average actual labor cost of the welding operation on part #319 performed in department #2. As was the case

Table 8

Means and Ranges of 25 Samples of 4 Taken from Welding Operation during November, 1968, Part #319, Department #2

Sample Number	Sample Mean ($\bar{X}$)	Sample Range (R)
1	\$2.48	\$0.24
2	2.62	.31
3	2.61	.26
4	2.32	.17
5	2.51	.20
6	2.29	.13
7	2.48	.29
8	2.29	.08
9	2.50	.26
10	2.48	.23
11	2.37	.18
12	2.47	.06
13	2.48	.21
14	2.42	.19
15	2.33	.29
16	2.50	.32
17	2.29	.26
18	2.56	.19
19	2.41	.37
20	2.44	.06
21	2.31	.23
22	2.28	.19
23	2.55	.26
24	2.29	.21
25	2.48	.34
	$\bar{\bar{X}}$ = \$2.43	$\bar{R}$ = \$0.22

with the blanking operation, each sample consisted of four observations with each observation representing the cost of welding one unit of part #319. Again, the basis for subgrouping was the order of production.

The mean of these 25 sample means was found to be

$$\bar{\bar{X}} = \frac{\sum \bar{X}}{k} = \frac{\$60.76}{25} = \$2.43$$

Because this figure was the preliminary estimate of the mean of the sampling distribution of the mean, it was used as the central line on the preliminary $\bar{X}$ chart shown in Figure 4.

The 3-sigma control limits were calculated on the basis of the average range ($\bar{R}$) and the A_2 factor previously discussed.[22] For the group of samples shown in Table 8 the average range was

$$\bar{R} = \frac{\sum R}{k} = \frac{\$5.53}{25} = \$0.22$$

Accordingly the preliminary control limits were placed at

$$UCL = \bar{\bar{X}} + A_2\bar{R} = \$2.43 + .73\ (\$.22) = \$2.59$$

$$LCL = \bar{\bar{X}} - A_2\bar{R} = \$2.43 - .73\ (\$.22) = \$2.27$$

These limits were used to complete the $\bar{X}$ chart shown in Figure 4.

In order to determine if all the November samples were taken from the same universe, the 25 sample means used in the above calculations were plotted on the preliminary $\bar{X}$ chart (Figure 4). An examination of this chart reveals that sample No. 2 and sample No. 3 exceeded the established control limits. Upon investigation the

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[22]See p. 60.

Preliminary Control Chart for Means of 25 Samples of 4 Taken
from Welding Operation during November, 1968,
Part #319, Department #2

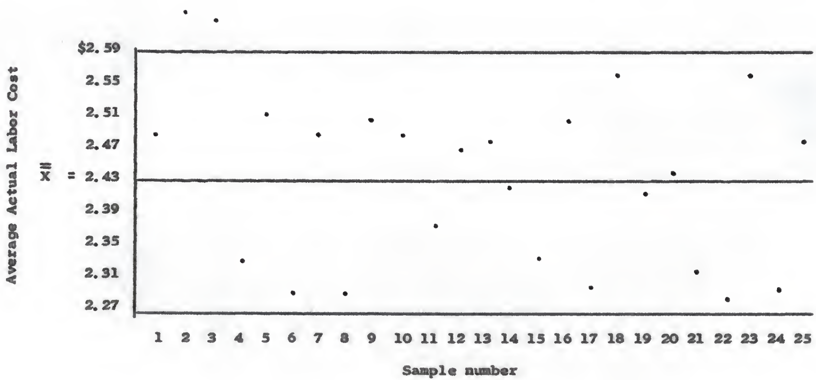

Figure 4

foreman identified a delay in the receipt of materials as the assignable cause. On the strength of the foreman's findings, the results of the two samples were dropped and two additional samples were added. The means and ranges of the revised group of samples appear in Table 9. The necessary calculations were then repeated using the data in this table. As shown in Figure 5, the central line on the revised $\overline{X}$ chart was $2.42 and the control limits were placed at $2.26 and $2.58. Because all the sample means fell inside the revised control limits, it was concluded that all sample observations were taken from the same universe.

Test of $\overline{X}$ Chart

Twenty-five additional samples of labor cost were taken from the welding operation during December, 1968, and the means of these samples were plotted on the revised $\overline{X}$ chart shown in Figure 6. These samples were taken in order to test the predictive ability of the $\overline{X}$ chart. If the estimates based on the November samples were correct, it should be possible to isolate assignable causes for all sample values outside the control limits. Figure 6 demonstrates that samples No. 2, No. 9, and No. 23 indicated the presence of assignable causes. As expected, the causes of these variations were identified. A problem with the welding fixture accounted for the variation in sample No. 2, while a malfunctioning welding gun explained samples No. 9 and No. 23.

Once again the propriety of the 3-sigma control limits selected for this study was tested by initiating a search for assignable causes in connection with two sample means (No. 4 and No. 6) which the $\overline{X}$ chart indicated were the result of random influences. The

Table 9

Means and Ranges of 25 Samples of 4 Taken from Welding Operation during November, 1968, Part #319, Department #2

Samples 2 and 3 Replaced

Sample Number	Sample Mean ($\overline{X}$)	Sample Range (R)
1	\$2.48	\$0.24
2	2.52	.26
3	2.47	.19
4	2.32	.17
5	2.51	.20
6	2.29	.13
7	2.48	.29
8	2.29	.08
9	2.50	.26
10	2.48	.23
11	2.37	.18
12	2.47	.06
13	2.48	.21
14	2.42	.19
15	2.33	.29
16	2.50	.32
17	2.29	.26
18	2.56	.19
19	2.41	.37
20	2.44	.06
21	2.31	.23
22	2.28	.19
23	2.55	.26
24	2.29	.21
25	2.48	.34
	$\overline{\overline{X}}$ = \$2.42	$\overline{R}$ = \$0.216

Revised Control Chart for Means of 25 Samples of 4 Taken
from Welding Operation during November, 1968,
Part #319, Department #2

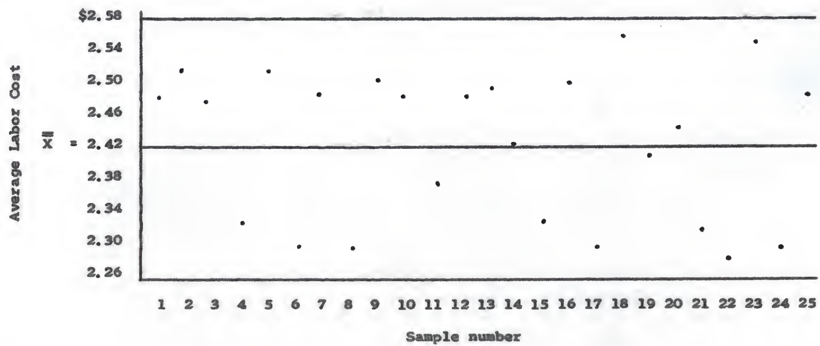

Figure 5

Control Chart for Means of 25 Samples of 4 Taken from Welding Operation during December, 1968, Part #319, Department #2

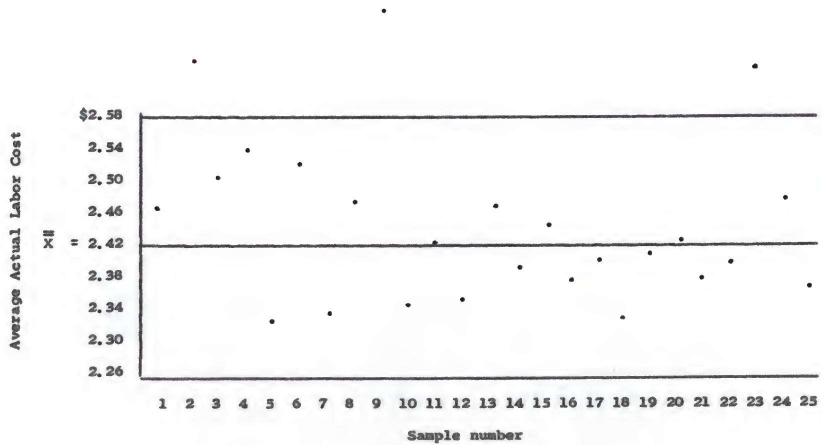

Figure 6

only factor uncovered was an experience differential on the part of the operators.

This discovery presented what was initially considered a serious problem. Should a separate control chart be used for experienced operators? Were there, in fact, two probability distributions of chance performance? When questioned, the foreman attributed this condition to the union seniority system. Consideration was then given to the possibility of constructing a control chart for each operator. However, the foreman disapproved of the idea because the operation in question was rotated among all the operators in the department. Another important consideration was possible union resistance to the use of individual control charts. Thus, a single chart by operation and part number was settled upon.

$\overline{X}$ Chart for Assembly Operation

The final probabilistic cost control model was developed for the assembly operation in Department #3. The object of interest in this department was the actual cost of assembling one unit of part #106. The procedure employed was similar to that used in the development of the two models previously discussed. Therefore, only the highlights of this activity will be cited.

The sample statistics derived from the 25 samples taken from the assembly operation during November, 1968, are presented in Table 10. An analysis of the data resulted in the construction of the $\overline{X}$ chart shown in Figure 7. A significant feature of the preliminary chart is the fact that all sample means fell within the preliminary control limits. As a result this chart was applied intact to December output.

Table 10

Means and Ranges of 25 Samples of 4 Taken from Assembly Operation during November, 1968, Part #106, Department #3

Sample Number	Sample Mean ($\overline{X}$)	Sample Range (R)
1	$2.99	$0.18
2	2.90	.30
3	3.01	.14
4	2.99	.09
5	2.90	.16
6	3.07	.11
7	3.10	.32
8	2.91	.13
9	3.09	.29
10	3.10	.28
11	2.97	.16
12	3.11	.27
13	2.94	.17
14	3.15	.31
15	2.90	.15
16	3.13	.30
17	3.04	.21
18	2.99	.08
19	2.90	.14
20	3.10	.31
21	2.95	.16
22	3.05	.21
23	2.97	.15
24	2.96	.18
25	3.14	.28
	$\overline{\overline{X}}$ = $3.01	$\overline{R}$ = $0.20

Preliminary Control Chart for Means of 25 Samples of 4 Taken from Assembly Operation during November, 1968, Part #106, Department #3

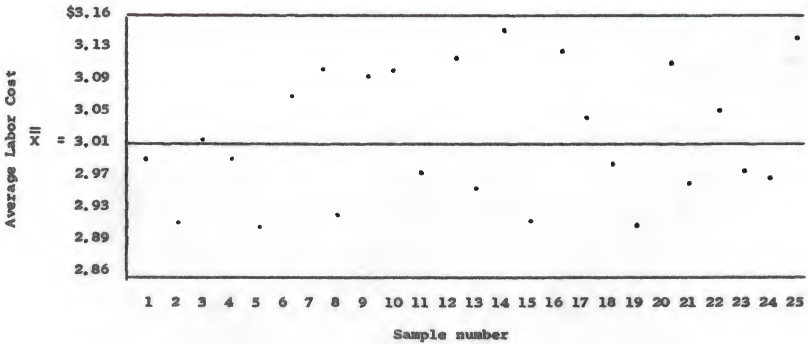

Figure 7

The means of the 23 additional samples taken during the month of December appear on the $\bar{X}$ chart reproduced in Figure 8. An investigation was undertaken on a current basis only in connection with sample No. 10 and sample No. 20; with No. 10, because it exceeded the upper control limit and with No. 20, because it afforded yet another opportunity to test the predictive ability of the established control limits. The assignable cause of the variation indicated by sample No. 10 was found to be a burr on one of the components. As indicated by sample No. 11, the prompt isolation of this assignable cause was instrumental in preventing the recurrence of a similar variation. The investigation of sample No. 20, on the other hand, was fruitless in the sense that no specific cause of the variation could be identified. Approximately two man-hours were spent in the investigation process by the foreman and budget analyst. Subsequently, several remaining possibilities were exhausted by the writer. In each case the result proved to be negative.

Effectiveness of $\bar{X}$ Charts

An inspection of Figures 3, 6, and 8 reveals that the $\bar{X}$ charts isolated six samples taken during the productive process as indicating the presence of assignable causes. An investigation conducted by the foreman and budget analyst on a current basis identified the cause of each of these sample values. On the basis of this experience it is possible to conclude that the $\bar{X}$ charts derived from sample data represent an efficient means of identifying those variations in performance which require investigation in the departments studied.

Control Chart for Means of 23 Samples of 4 Taken
from Assembly Operation during December, 1968,
Part #106, Department #3

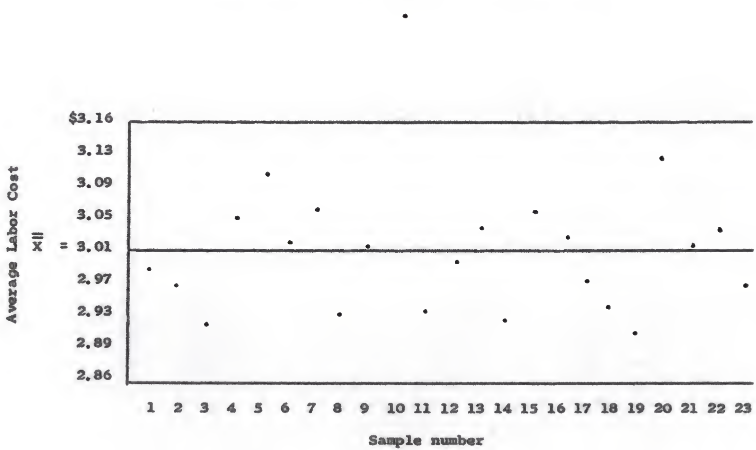

Figure 8

However, consideration was given to the fact that the control limits utilized in conjunction with the selected operations were unsatisfactory in the sense that they failed to identify additional variations in performance which were the result of assignable factors. In an effort to evaluate this possibility, those samples representing December performance which approached but still fell within the control limits were investigated. These included samples No. 2, No. 13, and No. 16 from the blanking operation; samples No. 4 and No. 6 from the welding operation; and sample No. 20 from the assembly operation. Even though all investigations were conducted immediately after the samples were taken, only one assignable cause was discovered. This was sample No. 16 -- Blanking, where the job was being run on a machine other than the one used when the job was originally timed.

These findings support the contention that the use of 3-sigma limits strikes an economic balance between the two errors of inference. More restrictive limits, such as 2-sigma, would lead to the investigation of random variations in performance.

CHAPTER V

ADJUSTMENT OF THE MODEL TO INCLUDE SUBJECTIVE PROBABILITIES

The initial reaction on the part of the foremen to the control chart proved to be fairly predictable. In spite of the fact that they had actively participated in the search for assignable causes associated with out-of-control sample points, they were unwilling to rely on the model as a variance investigation decision rule. They reasoned that, having been foremen in their departments for 20 years, they did not need probabilities to tell them when something was wrong. All they had to do was to walk out on the floor and they could tell if something was wrong.

The reaction was predictable because control devices of any kind are likely to meet with resistance. Horngren emphasized this very point in connection with budgets:

> Budgets place managers under the spotlight. The natural reaction to restriction, to criticism, and to control is resistance and self-defense. The job of education and selling is overwhelmingly important here. Too many department heads think that budgets represent a penny-pinching, negative brand of managerial pressure. To them, the word budget is about as popular as, say, lay-off, strike, or pay decrease. Ideally, company personnel should understand and accept the role of budgets as positive vehicles for company improvement,

department improvement, and individual improvement.[1]

In line with Professor Horngren's advice, a broader educational and selling job was attempted by the writer. The value of a variance investigation decision rule was stressed, as well as the use of the $\overline{X}$ chart to specify what is meant by "conditions of control." The importance of the control chart in establishing standard costs was also cited. Finally, the control chart was portrayed as a vehicle for the attainment of company objectives. However, in spite of this effort, the attitude of the foremen remained less than enthusiastic.

A sustained reaction of this nature would almost assuredly render probabilities ineffective as a vehicle for cost control. This follows from the fact that control charts are intended to be control devices at the operational level. Since operational responsibility is carried by the department foreman, his understanding and support of the charts are critical.

After a series of meetings with the foremen involved, it appeared that the major stumbling block continued to be their inability to understand the probabilistic technique. This was true in spite of an early meeting at which the statistical basis for the model was discussed. Therefore, in an effort to remedy this deficiency, "subjective probabilities" were introduced. There were two primary reasons for this inclusion:

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[1]Charles T. Horngren, Cost Accounting, A Managerial Emphasis (Englewood Cliffs, N. J.: Prentice-Hall Inc., 1967), p. 120.

1. To utilize a new approach in an effort to cultivate an understanding of the probabilistic technique, and
2. To involve the foremen directly in the actual construction of the individual $\bar{X}$ charts.

In each case, it was hoped the result would be an appreciation and an endorsement of the technique by the foremen.

Construction of $\bar{X}$ Chart Using Subjective Estimates - Blanking Operation

In the preceding chapter the $\bar{X}$ charts were constructed on the basis of the information obtained from 25 random samples of labor cost taken during November, 1968. More specifically, the sample information was used to estimate the parameters of the universe of individual performance. These parameters in turn were used to estimate the parameters of the sampling distribution of the mean from which the $\bar{X}$ charts were constructed.

Because the use of sample information in the construction of the $\bar{X}$ charts presented pedagogical difficulties, this approach was supplemented by the subjective technique described in this chapter. The distinguishing feature of the latter is the use of subjective estimates in the construction of the $\bar{X}$ charts. In essence, the foreman was asked to estimate the universe of individual random performance. This universe was then used to estimate the parameters of the sampling distribution. As was the case with the sample information, these estimated parameters were used to construct the $\bar{X}$ charts.

Initially, the foreman was asked to specify the range of individual performance if only chance factors were operative. The

estimate provided by the blanking department foreman is shown in Table 11. According to his estimate the actual labor cost of blanking 100 pieces of part #207 can be expected to vary between \$3.74 and \$4.10. From this information nine equal intervals were constructed. The foreman was then asked how 100 individual observations would be distributed within these intervals. The probabilities shown in Table 11 reflect this latter estimate. In addition, the foreman's estimate of the universe of individual performance is presented graphically in Figure 9.

The utilization of a single subjective estimate instead of 25 random samples necessitated a modification in the method of constructing the $\bar{X}$ chart. The mean of the foreman's estimated distribution was used as the central line on the $\bar{X}$ chart. This mean was calculated from the formula[2]

$$E(X) = \sum X\,P(X)$$

where X is the mid-point of each interval and P(X) is the probability that a particular value of X will occur. The actual calculations are shown in Table 12.

Because the foreman's estimate was considered to be the universe of individual performance, σ, the standard deviation of this universe, was calculated directly from the data in Table 11. As shown in Table 13, σ was obtained from the expression[3]

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[2]John Neter and William Wasserman, Fundamental Statistics for Business and Economics (3rd ed.; Boston: Allyn and Bacon, Inc., 1966), p. 209.

[3]F. E. Croxton, D. J. Cowden and S. Klein, Applied General Statistics (Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1967), p. 195.

Table 11

Foreman's Estimate of the Probability Distribution of Chance Performance

Blanking Operation

(Actual Labor Cost per Hundred Pieces)

Labor Cost	Probability
$3.74 but less than $3.78	.01
3.78 " " " 3.82	.15
3.82 " " " 3.86	.25
3.86 " " " 3.90	.20
3.90 " " " 3.94	.15
3.94 " " " 3.98	.10
3.98 " " " 4.02	.10
4.02 " " " 4.06	.03
4.06 " " " 4.10	.01

Graphical Presentation of Foreman's Estimated
Probability Distribution of Chance Performance

Blanking Operation

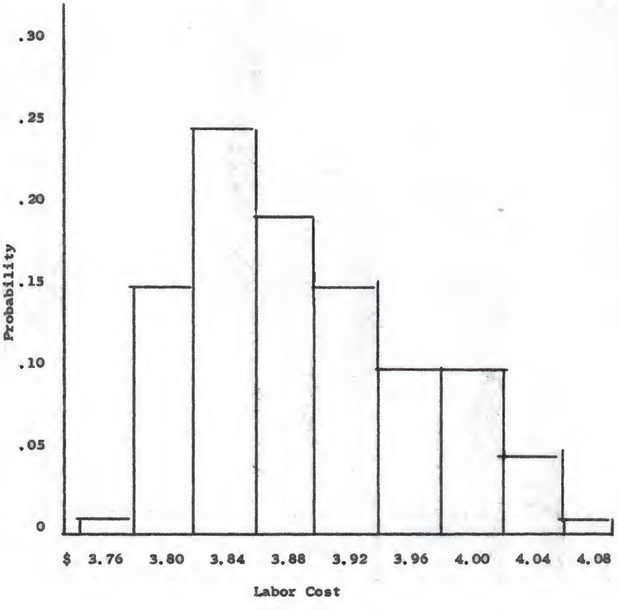


Figure 9

Table 12

Computation of Mean of Estimated Probability Distribution of Chance Performance

Blanking Operation

X	Pr(X)	X · Pr(X)
Mid-point of Interval	Probability of X	
\$3.76	.01	\$0.037
3.80	.15	.570
3.84	.25	.960
3.88	.20	.776
3.92	.15	.588
3.96	.10	.396
4.00	.10	.400
4.04	.03	.121
4.08	.01	.040
Total		$\overline{X}$ = \$3.89

Table 13

Computation of Standard Deviation of Estimated Probability Distribution of Chance Performance

Blanking Operation

X Mid - point of Interval	f Frequency	d Interval Deviation from Assumed Origin	fd	d^2	fd^2
\$3.76	1	-3	- 3	9	9
3.80	15	-2	-30	4	60
3.84	25	-1	-25	1	25
3.88	20	0	0	0	0
3.92	15	1	15	1	15
3.96	10	2	20	4	40
4.00	10	3	30	9	90
4.04	3	4	12	16	48
4.08	1	5	5	25	25
	100		24		312

$$\sigma = i\sqrt{\frac{\sum f(d)^2}{N} - \left(\frac{\sum fd}{N}\right)^2} = \$\ .0706$$

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{\$.0706}{2} = \$.035$$

$$\sigma = i \sqrt{\frac{\sum f (d)^2}{N} - \left(\frac{\sum f d}{N}\right)^2}$$

where d represents the deviation of the mid-pointof the interval from the assumed mean in terms of intervals, and i is the size of the interval.

With σ it was possible to estimate $\sigma_{\bar{X}}$ from the expression

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{\$.07}{2} = \$.035$$

This figure (\$.035) represented an estimate of the standard deviation of the sampling distribution of the mean on the assumption that an infinite number of samples of four observations were taken from the estimated universe of individual performance. As such it was used to construct the 3-sigma control limits as follows:

$$UCL = \bar{X} + 3\,\sigma_{\bar{X}} = \$3.89 + 3(\$.035) = \$3.995$$
$$LCL = \bar{X} - 3\,\sigma_{\bar{X}} = \$3.89 - 3(\$.035) = \$3.785$$

The $\bar{X}$ chart derived from the foreman's estimated probability distribution is shown in Figure 10. The means of the samples taken during December were then plotted on this chart.

The essential difference between the procedure here and the one presented in the preceding chapter is the use of subjective estimates. The end product in each case was a $\bar{X}$ chart for the blanking operation. However, in Chapter IV the chart was constructed on the basis of sample information obtained during November. In the subjective approach, the $\bar{X}$ chart was derived from the foreman's estimate. The significance of this procedure to foreman acceptance of

Control Chart Constructed on the Basis of Foreman's Estimated Probability Distribution of Chance Performance - Blanking Operation December Sample Means Plotted

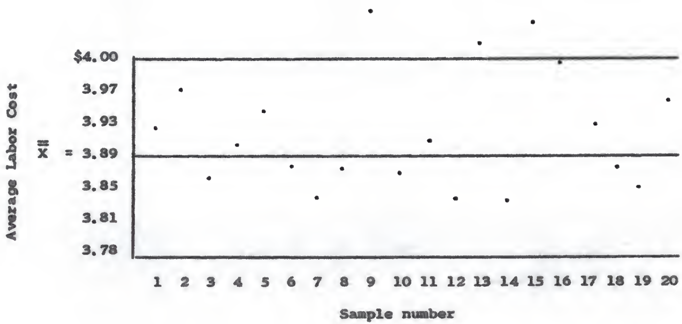


Figure 10

control charts will be appraised following a similar application to the welding and assembly operations.

Welding and Assembly Operations

The preceding discussion, which focused on the blanking operation, was somewhat detailed in an effort to clarify the distinguishing characteristics of the approach. In this section the estimates of the welding department foreman appear as Table 14 and Figure 11. The $\bar{X}$ chart based on these estimates appears in Figure 12. The same information with respect to the assembly operation is presented in Table 17 and Figures 13 and 14.

Comparison of $\bar{\bar{X}}$ Charts Derived from Sample Data and Foremen's Subjective Estimates

The ability of the $\bar{X}$ charts derived from sample data to predict the variation inherent in the associated operations was demonstrated in Chapter IV. If the $\bar{\bar{X}}$ charts based on the subjective estimates are to be useful they should not exhibit significant differences. Accordingly, the two control charts for each operation were compared on both practical and statistical grounds.

Foreman had seen stat limits before subjective est.

The two $\bar{\bar{X}}$ charts applicable to the blanking operation appear in Figure 15. Both charts indicate the presence of assignable causes in connection with sample No. 9 and No. 15. The reasons for these variations were, in fact, isolated at the time the samples were taken. In addition, the control chart derived from the foreman's estimate indicates that an out-of-control condition exists with respect to sample No. 13; yet a previous investigation of the point failed to explain the variation.

Table 14

Foreman's Estimate of the Probability Distribution of Chance Performance

Welding Operation

(Actual Labor Cost per Piece)

Labor Cost	Probability
\$2.20 but less than \$2.25	.02
2.25 " " " 2.30	.15
2.30 " " " 2.35	.20
2.35 " " " 2.40	.10
2.40 " " " 2.45	.15
2.45 " " " 2.50	.25
2.50 " " " 2.55	.10
2.55 " " " 2.60	.02
2.60 " " " 2.65	.01

Graphical Presentation of Foreman's Estimated Probability Distribution of Chance Performance

Welding Operation

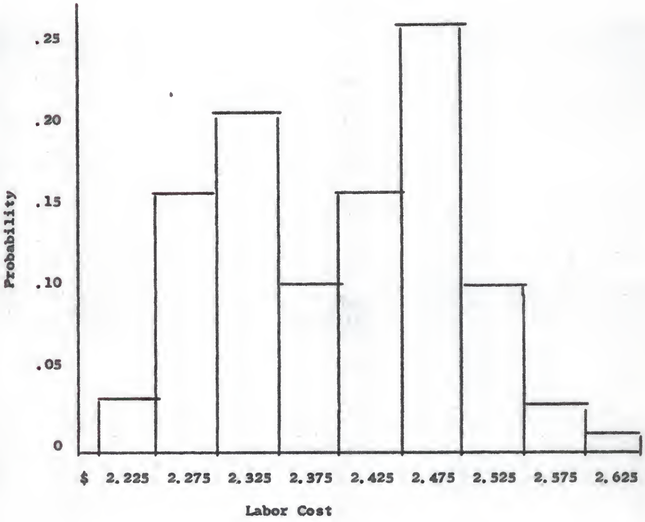


Figure 11

Table 15

Computation of Mean of Estimated Probability Distribution of Chance Performance

Welding Operation

X	Pr(X)	X · Pr(X)
Mid-point of Interval	Probability of X	
$2.225	.02	$0.044
2.275	.15	.341
2.325	.20	.465
2.375	.10	.237
2.425	.15	.363
2.475	.25	.618
2.525	.10	.252
2.575	.02	.052
2.625	.01	.026
Total		$\overline{X}$ = $2.39

Table 16

Computation of Standard Deviation of Estimated Probability Distribution of Chance Performance

Welding Operation

X Mid - point of Interval	f Frequency	d Interval Deviation from Assumed Origin	fd	d^2	fd^2
\$2.225	2	-3	- 6	9	18
2.275	15	-2	-30	4	60
2.325	20	-1	-20	1	20
2.375	10	0	0	0	0
2.425	15	1	15	1	15
2.475	25	2	50	4	100
2.525	10	3	30	9	90
2.575	2	4	8	16	32
2.625	1	5	5	25	25
	100		52		360

$$\sigma = i\sqrt{\frac{\sum f(d)^2}{N} - \left(\frac{\sum fd}{N}\right)^2} = \$\,.094$$

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{\$.094}{2} = \$\,.047$$

Control Chart Constructed on the Basis of Foreman's Estimated Probability Distribution of Chance Performance - Welding Operation December Sample Means Plotted

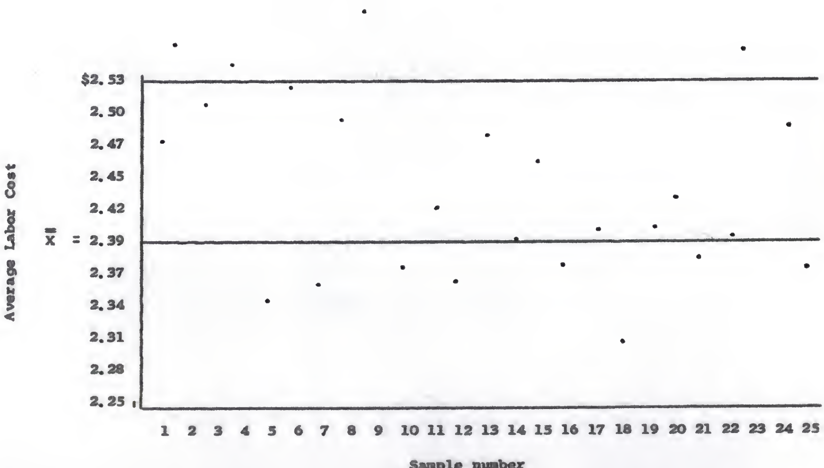


Figure 12

Table 17

Foreman's Estimate of the Probability Distribution of Chance Performance

Assembly Operation

(Actual Labor Cost per Piece)

Labor Cost	Probability
$2.80 but less than $2.85	.01
2.85 " " " 2.90	.03
2.90 " " " 2.95	.10
2.95 " " " 3.00	.20
3.00 " " " 3.05	.25
3.05 " " " 3.10	.20
3.10 " " " 3.15	.15
3.15 " " " 3.20	.05
3.20 " " " 3.25	.01

Graphical Presentation of Foreman's Estimated
Probability Distribution of Chance Performance

Assembly Operation

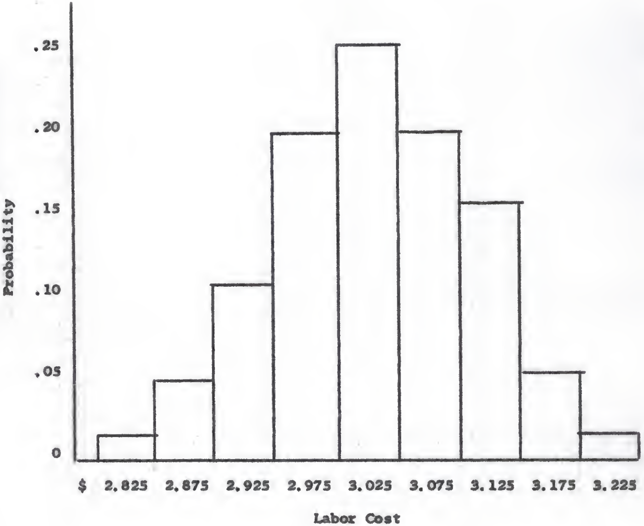


Figure 13

Table 18

Computation of Mean of Estimated Probability Distribution of Chance Performance

Assembly Operation

X Mid-point of Interval	Pr(X) Probability of X	X · Pr(X)
\$2.825	.01	\$0.028
2.875	.03	.086
2.925	.10	.292
2.975	.20	.595
3.025	.25	.756
3.075	.20	.615
3.125	.15	.468
3.175	.05	.158
3.225	.01	.032
		$\overline{X}$ = \$3.03

Table 19

Computation of Standard Deviation of Estimated Probability Distribution of Chance Performance

Assembly Operation

X Mid - point of Interval	f Frequency	d Interval Deviation from Assumed Origin	fd	d^2	fd^2
\$2.825	1	-4	- 4	16	16
2.875	3	-3	- 9	9	27
2.925	10	-2	-20	4	40
2.975	20	-1	-20	1	20
3.025	25	0	0	0	0
3.075	20	1	20	1	20
3.125	15	2	30	4	60
3.175	5	3	15	9	45
3.225	1	4	4	16	16
	100		16		244

$$\sigma = i \sqrt{\frac{\sum f(d)^2}{N} - \left(\frac{\sum fd}{N}\right)^2} = \$ .078$$

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \$ .039$$

Control Chart Constructed from Foreman's Estimated Probability
Distribution of Chance Performance - Assembly Operation
December Sample Means Plotted

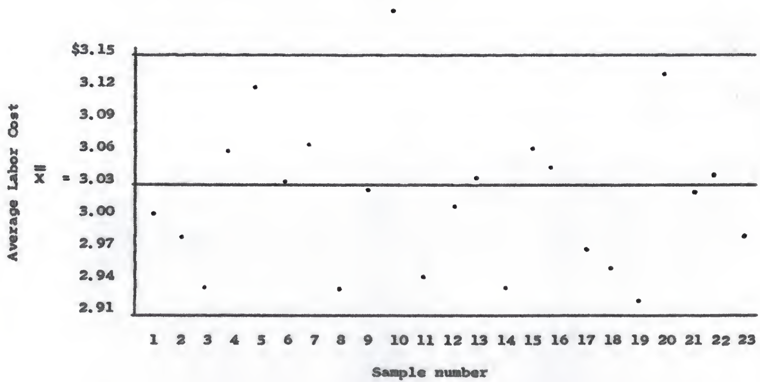


Figure 14

Comparison of Control Charts Derived from Sample Data and Foreman's Estimate
December Samples on Blanking Operation

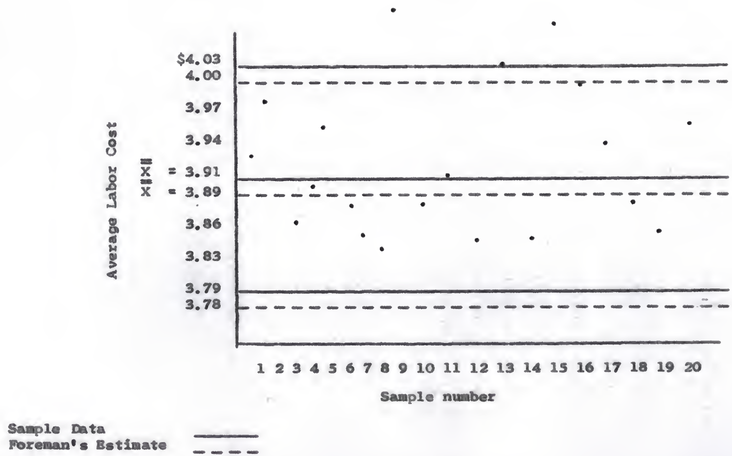


Figure 15

A similar comparison is made in Figure 16 for the welding operation. Both charts isolate samples No. 2, No. 9, and No. 23; the causes of which have been previously indicated. Again, however, the foreman's chart isolated one point (No. 4) for which an earlier investigation failed to identify an assignable cause. In the final comparison (Figure 17), both charts highlight sample No. 10.

An examination of figures 15, 16, and 17 reveals that in each case the $\overline{X}$ chart based on the foreman's subjective estimate was tighter in the sense that the 3-sigma limits were closer to the central line. This, of course, reflects a more conservative estimate of the standard deviation of the universe of individual performance. As shown in Table 20, all three foremen believed the dispersion to be less than that indicated by the sample data. Consequently, when these charts were utilized in conjunction with December operations they showed two more points out of control than was true of the charts derived from the sample data.

Table 20

Estimates of the Mean and Standard Deviation of the Chance Distribution of Individual Performance

Operation	Estimated Parameter	Foreman's Estimate	Sample Data
Blanking	Mean	\$3.89	\$3.91
	Standard Dev.	.071	.078
Welding	Mean	\$2.39	\$2.42
	Standard Dev.	.094	.105
Assembly	Mean	\$3.03	\$3.01
	Standard Dev.	.078	.097

Comparison of Control Charts Derived from Sample Data and Foreman's Estimate
December Samples on Welding Operation

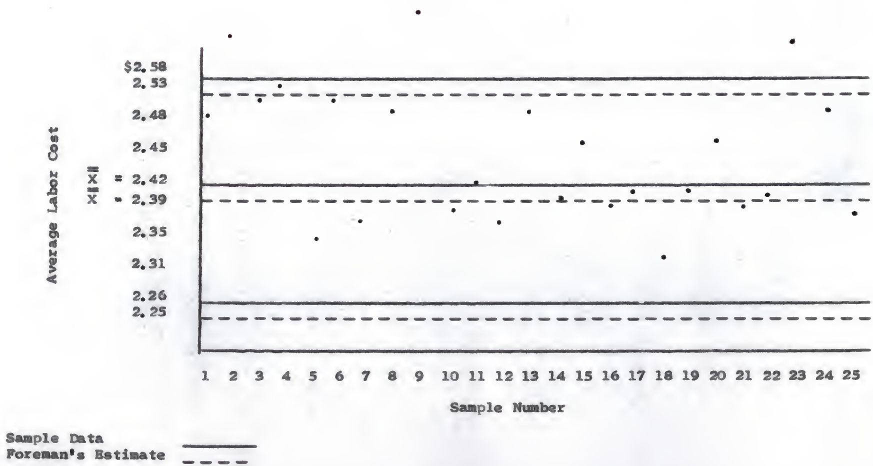
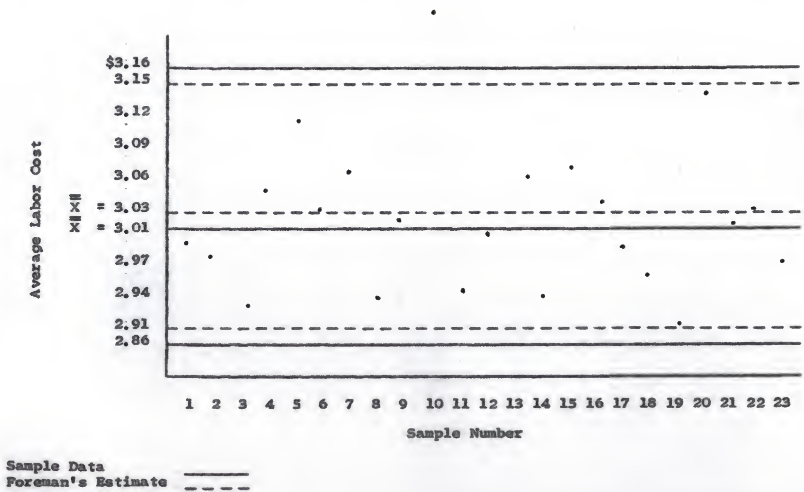


Figure 16

Comparison of Control Charts Derived from Sample Data and Foreman's Estimate
December Samples on Assembly Operation



Statistical Comparison of Procedures Employed

The difference between the standard deviation computed from the foremen's estimate and the estimate of the standard deviation of the universe of individual performance obtained from the sample data was then tested for each operation to determine if these differences were statistically significant. The question was raised whether the samples could have been drawn from the foremen's universe. The test was accomplished by using the "F" distribution and comparing the ratio of the computed variances (σ_1^2, σ_2^2) with the critical value of F.

> The distribution of F is different for each combination of numbers of degrees of freedom: $n_1 - 1$ and $n_2 - 1$. In this sense ... there is really a family of F-distributions; and, in order to define the desired member of this family of F-distributions, it is necessary to specify two numbers, the two values of degrees of freedom....[4]

In Chapter IV, random samples of four were utilized to estimate the standard deviation of the universe of individual performance. Thus, the value of $n_1 - 1 = 3$. Because each foreman's estimate was considered to be the universe of individual performance, $n_2 - 1 = \infty$. At the .02 level of significance the critical value[5] of F (3, ∞) was found to be 3.78. For the individual operations the ratios of the universe variances were as follows:

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[4]Samuel B. Richmond, Statistical Analysis (New York: The Ronald Press, 1964), p. 307.

[5]Ibid., p. 580.

$$\text{Blanking} \quad \frac{.078^2}{.071^2} = \frac{.0061}{.0050} = 1.22$$

$$\text{Welding} \quad \frac{.105^2}{.094^2} = \frac{.0110}{.0088} = 1.25$$

$$\text{Assembly} \quad \frac{.097^2}{.078^2} = \frac{.0094}{.0061} = 1.54$$

Because the critical value of F exceeded each of the computed ratios, the hypothesis that the universe variance (or standard deviation) has the value specified by the foreman's estimate was not rejected. The same conclusion was reached when the hypothesis was tested at the .10 level of significance, where the critical value[6] of F is 2.60.

The fact that the hypothesis was not rejected for any of the operations indicates that from a statistical point of view either method of constructing $\bar{\bar{X}}$ charts is acceptable. By affirming that the actual samples could have come from the foreman's universe, the "F" test demonstrated the statistical equivalence of the two approaches employed in this study. The test also served to validate the assumption that the foreman's estimate was the universe of individual performance. Finally, these statistical conclusions were supported empirically when the $\bar{\bar{X}}$ charts based on the foremen's estimates isolated only two performance values for which assignable causes could not be found.

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[6]*Ibid.*, p. 580.

Importance of Estimation Procedure

The most obvious benefit stemming from the use of the estimation procedure was the increased support of the probabilistic technique on the part of the foremen. This support appeared to be the result of two related factors: participation and understanding. By actively participating in the formulation of the model, the foremen developed a vested interest in it. This interest, in turn, was accompanied by a fuller understanding of the technique and its objectives. The result was a measurable change in the level of their support.

The importance of this support cannot be overemphasized because the success of the probabilistic model is in large part dependent upon the acceptance accorded it by lower level management. This follows from the fact that the model is designed to facilitate control at the operational level. Therefore, to be effective it must be endorsed by the persons directly responsible for these operations -- the foremen.

One of the findings emanating from this research is that operating costs cannot be controlled simply by means of a directive from middle management. The concept of a probabilistic cost control model was readily endorsed by middle management at the beginning of this study. However, the control possibilities were not realized until the men directly responsible for operating costs had endorsed the approach.

The fact that the initial $\bar{\bar{X}}$ charts derived from subjective estimates lacked some of the predictive power demonstrated by the charts based on sample data may be considered a necessary cost of

implementing probabilistic controls. It is a cost because some random performance is likely to be investigated; but this possibility was also present when using sample data. Therefore, the cost is minor relative to the benefits derived. These include endorsement of the technique at an early stage and a smooth transition to the probabilistic approach through the conversion of subjective estimates into probability control limits. Finally, this cost is minor because it is short-lived. Regardless of how the control limits are initially established (sample data or subjective estimates), they can be revised as additional information becomes available.

CHAPTER VI

EVALUATION OF PROBABILISTIC CONTROLS EMPLOYED

Attempts to make probabilistic controls operational for three selected operations at the A. O. Smith Corporation were detailed in Chapters IV and V. In this chapter the following criteria are utilized as a means of evaluating these controls:

1. The control system provides a conceptual basis to aid in distinguishing between significant and insignificant variations in performance.
2. The control system provides a basis for appraising reported variations.
3. The control system will have a favorable impact on cost control procedure.

Each of these criteria is applied to both the A. O. Smith Corporation's traditional standard labor cost control system and the system of probabilistic controls.

Conceptual Basis

As shown in Table 1,[1] the A. O. Smith Corporation's labor cost control system in existence at the time this study was begun reports every departure from standard. From this it might be inferred that all such variances were considered significant in the sense that they signal the need for consideration and investigation. The fact that all short-run deviations are reported is cited by Zannetos as presumptive evidence that they require explanation.[2] However, as was pointed out in Chapter II, all reported variances are not, as a matter of fact, investigated, which indicates that some variations are considered insignificant. The question left unresolved is the manner in which a distinction is made at the conceptual level between significant and insignificant variances.

In the probabilistic model utilized, this distinction was made by virtue of the explicit recognition of the role of chance in performance. It is generally recognized in quality control that performance varies for unexplainable reasons.

> This chance variation is the sum of the effects of the whole complex of chance causes. In this complex of causes the effect of each cause is slight, and no major part of the total variation can be traced to a single cause.[3]

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[1]See p. 15.

[2]Z. S. Zannetos, "Standard Costs as a First Step to Probabilistic Control: A Theoretical Justification, an Extension and Implications," The Accounting Review, Vol. XXXIX (April, 1964), p. 297.

[3]Acheson J. Duncan, Quality Control and Industrial Statistics (Homewood, Illinois: Richard D. Irwin, Inc., 1965), p. 337.

Because of chance, variability in performance is considered inevitable. Short of revising a process, very little can be done to eliminate such variation.

When the concept of chance was introduced in the probabilistic model, the result was explicit recognition of the fact that costs will exhibit the type of variation observed in random sampling from a stable population. In other words, costs are subject to forces similar to those which cause a coin to turn up heads or tails when flipped in a random manner. As a result, actual costs will rarely be equal to standard costs. Some variances are to be expected. The basic reason for such variation is that "Some stable 'system of chance causes' is inherent in any particular scheme of production . . . ."[4]

It is the realization that variation in performance as well as costs is inevitable which provides the conceptual basis for distinguishing between significant and insignificant cost variances. Those variances caused by chance factors are insignificant because:

> There is no point in seeking special causes for individual cases, because random variations are beyond management's ability to regulate or eliminate.[5]

On the other hand, those variances which fall outside the limits due to chance are significant. Assignable causes for such variances may be identified and eliminated. However, this conceptual distinction is insufficient as a guide to action. A variance investigation

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[4]Eugene L. Grant, Statistical Quality Control (New York: McGraw-Hill Book Company, Inc., 1954), p. 1.

[5]Charles T. Horngren, Cost Accounting: A Managerial Emphasis (Englewood Cliffs, N. J.: Prentice-Hall Inc., 1967), p. 805.

decision rule must incorporate criteria which serve to appraise the significance of reported variations.

Basis for Appraising Reported Variations

To the foremen and budget analysts at the A. O. Smith Corporation a significant variance is one which requires investigation. Because the role of chance is not explicitly recognized in the traditional standard cost system employed, they rely on experience and judgment when deciding whether to investigate reported variances. Horngren indicates that this approach is widespread:

> Management tends to use judgment in deciding whether or not a variance on a given item deserves investigation .... Guesses or hunches are fundamental parts of managerial behavior; yet these subjective methods often engender management disagreements and barren investigations.[6]

During the course of this study, it became clear that each foreman relied on differing subjective criteria when making the investigation decision. One foreman first directed his attention to the net variance computed for his department. If this figure did not appear to be out of line with previously reported amounts, the individual variances were ignored. To a large extent this approach explains the apparent indifference toward reported individual variances. It does, however, presuppose the formulation of criteria to identify an "out-of-line" condition. Still other foremen made the investigation decision after examining individual reported variances. However, they did not agree on the amount of acceptable variation. For example, one of these foremen considered 2 per cent of

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[6] Ibid., p. 802.

standard significant, while another maintained that 10 per cent was a better measure of significance.

When the concept of chance is introduced, as was done in the probabilistic model, a significant variance is defined as one which requires investigation. Its significance, however, is derived from statistical evidence concerning the presence of assignable causes. Similarly, a recognition of the operation of chance defines insignificant variances. In order to distinguish between variances having assignable causes and variances which are random in nature, it is necessary to specify the limits within which chance is operative. The significance of a reported variation in performance may then be evaluated in terms of its probability of chance occurrence. In this way intuitive measures of significance are replaced by objective criteria. The logic underlying this use of probabilities was presented previously:

> Stated succinctly, chance affects some variance classifications; probability statistics evaluates patterns of chance occurrences; therefore, probability statistics is useful to analyze those variances affected by chance.[7]

Although the probability of chance occurrence was not calculated for each variation in performance observed at the A. O. Smith Corporation, this information could have been readily obtained from the probability distribution of chance performance identified in connection with each operation. For example, in this study the sample data obtained from the blanking operation during November defined this distribution as having a mean of $3.91 and a standard

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[7]R. W. Koehler, "The Relevance of Probability Statistics to Accounting Variance Control," Management Accounting, Vol. L (October, 1968), p. 37.

deviation of \$.04. If it is assumed that a sample value of \$3.93 was subsequently obtained, the probability that a value this large or larger was due to chance alone is 31 per cent. This probability was obtained by computing the number of standard deviations that \$3.93 is from the mean, \$3.91.

$$\text{Number of standard deviations} = \frac{\text{sample value - mean of chance distribution}}{\text{standard deviation of chance distribution}}$$

$$= \frac{\$3.93 - \$3.91}{\$.04} = .5$$

In a table of normal probabilities it was found that the probability of a sample value being .5 or more standard deviations to the right of the mean is 31 per cent.

With this analysis, the probability that any sample value was caused by random factors may be computed. The investigation decision may then be made on the basis of the computed probability.

In this study, however, individual probabilities were not computed for each sample value because these probabilities were incorporated in the control limits. When the control limits were placed a distance of 3 standard deviations on either side of the mean, 99.73 per cent of the sample means were included in this interval. Thus, if a sample value fell outside the control limits, the probability that it was due to chance was less than .0027 per cent. The obvious advantage of building individual probabilities into the control limits is that it minimized the amount of statistical orientation required for factory personnel.

However, the utilization of probabilities in the formulation of a variance investigation decision rule introduced the possibility

of two types of error. The first of these is the investigation of random performance which is known as Type I error. Because 3-sigma control limits were used in this study, the probability of Type I error was .0027. This is based on the assumption that the sample means were normally distributed. The selection of these limits was designed to minimize the possibility of investigating chance performance. From an empirical standpoint this objective was attained. For the three operations an investigation of each sample value which fell outside the control limits resulted in the identification of an assignable cause.

The second type of error is the failure to investigate assignable performance (Type II). Unlike Type I error, the probability of Type II error cannot be directly evaluated because "a Type II error is unique to some particular alternative non-chance population with a specified distribution and mean."[8] Thus, there is a unique distribution of performance values associated with each assignable cause. Each such distribution, in turn, has a separate probability of Type II error. Thus, in order to determine the probability of a Type II error, it is necessary to identify both the assignable cause present at the time of the computation and the distribution associated with that cause.

Because available data did not permit the estimation of the non-chance distributions, the susceptibility of 3-sigma control limits to Type II error was tested empirically. As indicated in Chapter IV,[9] a number of sample values which approached but were

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[8] Ibid., p. 40.

[9] See pp. 79 and 81.

still inside the 3-sigma control limits were investigated for each operation. The objective was to determine if an assignable cause could be found for a sample value which the $\overline{X}$ chart indicated was due to chance (Type II error). For the three operations described, only one such sample value was discovered. From the results of this study it would appear, therefore, that the 3-sigma control limits do strike a balance between the two types of error.

Favorable Impact on Cost Control Procedure

Up to this point the discussion has emphasized the conceptual basis and the criteria utilized to evaluate the significance of individual variations in performance. However, the implications for cost control of the procedures employed also merit consideration.

In the traditional labor cost control system used at the A. O. Smith Corporation, the weekly "labor performance" report[10] shows the dollar variance as a percentage of standard for each operation-part number combination. This report, which is compiled every Friday for the preceding week, applies to performance which took place anywhere from seven to eleven days earlier. As a result, the matter of timeliness becomes important.

Ideally, variances should be investigated as soon after their reporting as possible in order to minimize the period of cost disruption. However, reliance upon reports prepared periodically allows variances to accumulate. Because of this, in the event an investigation is subsequently conducted, the likelihood of its success is lessened. Experience has demonstrated that the more time

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[10]See p. 15.

which elapses between the occurrence of a variance and its investigation, the more difficult it becomes to isolate the reason for the variance. Even if a diligent search should uncover the cause, the cost of the investigation is likely to be increased because of the additional time spent in the investigative process.

The weekly "labor performance" report also indicates that a net variance is computed for the department as a whole. By their own admission, some foremen direct their attention to this composite figure. This would tend to indicate that they attempt to control costs at the departmental rather than at the operational level. However, in fact, the department represents a grouping of operations, which gives rise to the possibility that significant favorable and unfavorable operational variances will be offset in a summarized reporting of variances by departments. Thus, by virtue of aggregation, opportunities for cost reduction may be overlooked.

The probabilistic controls utilized in this study relied on random sampling of specific operations during the productive process to identify the presence of variances having assignable causes. In the event such a condition was detected by the daily samples, a search for the cause was initiated immediately. This procedure maximized the likelihood of isolating the assignable cause and minimized the period during which the cause was operative, thereby exerting control before the variances accumulated. In addition, the use of random samples presupposed the specification of the probability distribution of chance performance associated with each operation. From this it follows that control was exercised at the operational level.

Finally, the use of random sampling to detect the presence of assignable causes suggests a modification of the reporting procedure. Under the existing system, all variances are reported without any indication of their significance. With the use of random sampling it would be possible to report all sample results daily, or for a lesser period, along with an indication of their probabilistic significance. By the use of symbols or colors the various levels of management could be informed as to which performance measures exceeded the control limits. A further simplification would be the reporting of only out-of-control sample values. In this way the attention and effort of management would be directed to the investigation of only probabilistically significant variations in performance.

CHAPTER VII

SUMMARY AND CONCLUSION

This study was prompted by suggestions made in the accounting literature that probabilities be utilized in cost control. In general, the recommendations were to incorporate probabilities into the standard cost system in order to formulate a decision rule to be used in connection with variance analysis. The function of the decision rule is to evaluate performance and thereby to identify those operations currently out of control. In this way the traditional approach to cost control, which isolates historical discrepancies, could be replaced by a mathematical technique which has the advantage of identifying problem areas on a current basis. Consequently, the attention and effort of management would be directed to the investigation of only those variations in performance which were probabilistically significant.

In spite of these recommendations, very few references to the actual utilization of probabilistic controls appear in the accounting literature. Accordingly, this study was undertaken in an effort to explore the feasibility of utilizing probabilistic cost controls in a manufacturing firm, specifically, the A. O. Smith Corporation, Milwaukee, Wisconsin.

During the course of the study, four probabilistic control models were considered for possible use. These included:

1. The control chart for sample means ($\overline{X}$ chart) developed in the area of quality control.
2. The Controlled Cost model developed by Luh.
3. The Bierman, Fouraker, and Jaedicke model.
4. The modern decision theory model as presented by Onsi.

After carefully examining each model and the data available at the A. O. Smith Corporation, the writer selected the control chart for sample means for use in experimentation. At the heart of this selection was the conviction that the control chart approach is the logical starting point in an attempt to utilize probabilistic controls. Available data did not permit the estimation of variables called for in the other models. Management personnel were equally unable to quantify these variables. The only remaining possibility was to make a number of simplifying assumptions. However, it quickly became apparent that the utility of the models would be impaired by the ensuing estimates. In the case of the $\overline{X}$ charts, however, the necessary data could be obtained. In addition, the potential for widespread application of this technique was recognized along with the possibility of using the control chart as an evolutionary vehicle.

In order to implement this probabilistic control model, three manufacturing operations were selected: blanking, welding, and assembly. During November, 1968, 25 random samples of labor cost were taken from each of the operations. This sample information was then used to construct the $\overline{X}$ charts. The purpose of the charts was to distinguish between random and assignable performance. To

accomplish this objective the control limits were placed so as to include 99.73 per cent of all random performance measures. Underlying this percentage is the assumption that the sample means were normally distributed. As a result, any sample value falling outside the control limits was considered to be an indication of the presence of assignable causes.

The predictive ability of the resulting $\overline{X}$ charts was subsequently tested during December, 1968. A number of additional random samples of labor cost were taken in this period, and the mean of each of these samples was plotted on the $\overline{X}$ chart. A search for assignable causes was initiated immediately in connection with those sample means which fell outside the previously established control limits. In all, six such sample values were investigated, and an assignable cause was isolated in each case. Thus, no Type I error had been made.

Consideration was also given to the possibility that the $\overline{X}$ charts had failed to identify other assignable performance, that is, the question of Type II error was raised. To evaluate this possibility empirically, several of the more extreme sample values which the $\overline{X}$ chart attributed to chance were investigated. Of the six such investigations undertaken, only one resulted in the identification of an assignable cause.

Despite the apparent ability of the $\overline{X}$ charts to distinguish between random and assignable performance, the foremen at the A. O. Smith Corporation did not readily endorse the probabilistic technique. Their support was considered essential because the $\overline{X}$ charts were intended to facilitate cost control at the operational level. In an effort to overcome this reluctance to rely upon the output of

the model, a different approach to the construction of $\bar{X}$ charts was introduced by the writer. Instead of using November sample information to construct the charts, subjective estimates of the foremen were utilized. In each case the end product was a $\bar{X}$ chart, but the use of subjective estimates directly involved the foremen in their construction. By facilitating the transition from personal beliefs to probabilistic models, this involvement stimulated the interest of the foremen in probabilistic controls. The result was a marked increase in the level of support accorded the $\bar{X}$ charts by the foremen. When the differences between the $\bar{X}$ charts derived from sample data and those derived from subjective estimates were tested, these differences were not found to be statistically significant.

Thus, the results of this limited study indicate the feasibility of utilizing probabilities for labor cost control in the standard cost system of a manufacturing firm. In addition, the following conclusions emerge:

1. By incorporating the concept of chance, the control system provides a conceptual basis to aid in distinguishing between significant and insignificant variations in performance.
2. Through the probability of chance occurrence, the control system provides a basis for appraising reported variations.
3. The control system will have a favorable impact on cost control procedure. Random sampling during the productive process facilitates the identification of the presence of assignable

causes on a current basis. In addition, the $\overline{X}$ chart operates as a cost control vehicle at the operational level, thereby eliminating the problem of aggregation associated with a summarized reporting of variances by departments.

4. 3-sigma control limits appear to strike a satisfactory balance between Type I and Type II errors.
5. Both sample data and subjective estimates may be utilized in the construction of $\overline{X}$ charts. However, in this study the use of subjective estimates promoted the understanding and acceptance of the probabilistic technique by the foremen and facilitated the transition from traditional to probabilistic controls.

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BIOGRAPHICAL SKETCH

Frank Robert Probst was born October 4, 1940, at Milwaukee, Wisconsin. In June, 1958, he was graduated from Marquette University High School. In June, 1964, he received the degree of Master of Business Administration from Marquette University. In 1966 he enrolled in the Graduate School of the University of Florida, working toward the degree of Doctor of Philosophy. In September, 1968, he returned to Marquette University where he holds the rank of Assistant Professor of Accounting.

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This dissertation was prepared under the direction of the chairman of the candidate's supervisory committee and has been approved by all members of that committee. It was submitted to the Dean of the College of Business Administration and to the Graduate Council, and was approved as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

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